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Charmed Baryons

prepared by Hai-Yang Cheng

Contents

I. INTRODUCTION

In the past years many new excited charmed baryon states have been discovered by BaBar, Belle and CLEO. In particular, B factories have provided a very rich source of charmed baryons both from B decays and from the continuum $e^+e^- \to c\bar{c}$. A new chapter for the charmed baryon spectroscopy is opened by the rich mass spectrum and the relatively narrow widths of the excited states. Experimentally and theoretically, it is important to identify the quantum numbers of these new states and understand their properties. Since the pseudoscalar mesons involved in the strong decays of charmed baryons are soft, the charmed baryon system offers an excellent ground for testing the ideas and predictions of heavy quark symmetry of the heavy quark and chiral symmetry of the light quarks.

The observation of the lifetime differences among the charmed mesons D^+, D^0 and charmed baryons is very interesting since it was realized very early that the naive parton model gives the same lifetimes for all heavy particles containing a heavy quark Q, while experimentally, the lifetimes of Ξ_c^+ and Ω_c^0 differ by a factor of six ! This implies the importance of the underlying mechanisms such as W-exchange and Pauli interference due to the identical quarks produced in the heavy quark decay and in the wavefunction of the charmed baryons. With the advent of heavy quark effective theory, it was recognized in early nineties that nonperturbative corrections to the parton picture can be systematically expanded in powers of $1/m_Q$. Within the QCD-based heavy quark expansion framework, some phenomenological assumptions can be turned into some coherent and quantitative statements and nonperturbative effects can be systematically studied.

Contrary to the significant progress made over the last 20 years or so in the studies of the heavy meson weak decay, advancement in the arena of heavy baryons is relatively slow. Nevertheless, the experimental measurements of the charmed baryon hadronic weak decays have been pushed to the Cabibbo-suppressed level. Many new data emerged can be used to test a handful of phenomenological models available in the literature. Apart from the complication due to the presence of three quarks in the baryon, a major disparity between charmed baryon and charmed meson decays is that while the latter is usually dominated by factorizable amplitudes, the former receives sizable nonfactorizable contributions from W-exchange diagrams which are not subject to color and helicity suppression. Besides the dynamical models, there are also some considerations based on the symmetry argument and the quark diagram scheme.

The exclusive semileptonic decays of charmed baryons like $\Lambda_c^+ \to \Lambda e^+(\mu^+)\nu_e$, $\Xi_c^+ \to \Xi^0 e^+\nu_e$ and $\Xi_c^0 \to \Xi^- e^+\nu_e$ have been observed experimentally. Their rates depend on the heavy baryon to the light baryon transition form factors. Experimentally, the only information available so far is the form-factor ratio measured in the semileptonic decay $\Lambda_c \to \Lambda e\bar{\nu}$.

Although radiative decays are well measured in the charmed meson sector, e.g. $D^* \to D\gamma$ and $D_s^+ \to D_s^+\gamma$, only three of the radiative modes in the charmed baryon sector have been observed, namely, $\Xi_c'^0 \to \Xi_c^0\gamma$, $\Xi_c'^+ \to \Xi_c^+\gamma$ and $\Omega_c^{*0} \to \Omega_c^0\gamma$. Charm flavor is conserved in these electromagnetic charmed baryon decays. However, it will be difficult to measure the rates of these decays because these states are too narrow to be experimentally resolvable. There are also charm-flavor-conserving weak radiative decays such as $\Xi_c \to \Lambda_c\gamma$ and $\Omega_c \to \Xi_c\gamma$. In these decays, weak radiative transitions arise from the diquark sector of the heavy baryon whereas the heavy quark behaves as a "spectator". The charm-flavor-violating weak radiative decays, e.g., $\Lambda_c^+ \to \Sigma^+\gamma$ and $\Xi_c^0 \to \Xi^0\gamma$, arise from the W-exchange diagram accompanied by a photon emission from the external quark.

Two excellent review articles on charmed baryons can be found in [1, 2].

II. PRODUCTION OF CHARMED BARYONS AT BESIII

Production and decays of the charmed baryons can be studied at BESIII once its center-of-mass energy $\sqrt{s}$ is upgraded to the level above 4.6 GeV. In order to estimate the number of charmed baryon events produced at BESIII, it is necessary to know its luminosity, the cross section $\sigma(e^+e^- \to c\bar{c})$ at the energies of interest and the fragmentation function of the c quark into the charmed baryon. ....

III. SPECTROSCOPY

Charmed baryon spectroscopy provides an ideal place for studying the dynamics of the light quarks in the environment of a heavy quark. The charmed baryon of interest contains a charmed quark and two light quarks, which we will often refer to as a diquark. Each light quark is a triplet of the flavor SU(3). Since $\mathbf{3}\times\mathbf{3} = \bar{\mathbf{3}}+\mathbf{6}$, there are two different SU(3) multiplets of charmed baryons: a symmetric sextet $\mathbf{6}$ and an antisymmetric antitriplet $\bar{\mathbf{3}}$. For the ground-state s-wave baryons in the quark model, the symmetries in the flavor and spin of the diquarks are correlated. Consequently, the diquark in the flavor-symmetric sextet has spin 1, while the diquark in the flavor-antisymmetric antitriplet has spin 0. When the diquark combines with the charmed quark, the sextet contains both spin 1/2 and spin 3/2 charmed baryons. However, the antitriplet contains only spin 1/2 ones. More specifically, the Λ_c^+, Ξ_c^+ and Ξ_c^0 form a $\bar{\mathbf{3}}$ representation and they all decay weakly. The Ω_c^0, $\Xi_c'^+$, $\Xi_c'^0$ and $\Sigma_c^{++,+,0}$ form a $\mathbf{6}$ representation; among them, only Ω_c^0 decays weakly. Note that we follow the Particle Data Group (PDG) [3] to use a prime to distinguish the Ξ_c in the $\mathbf{6}$ from the one in the $\bar{\mathbf{3}}$.

The lowest-lying orbitally excited baryon states are the p-wave charmed baryons with their quantum numbers listed in Table I. Although the separate spin angular momentum S_ℓ and orbital angular momentum L_ℓ of the light degrees of freedom are not well defined, they are included for guidance from the quark model. In the heavy quark limit, the spin of the charmed quark S_c and the total angular momentum of the two light quarks $J_\ell = S_\ell + L_\ell$ are separately conserved. It is convenient to use them to enumerate the spectrum of states. There are two types of $L_\ell = 1$ orbital excited charmed baryon states: states with the unit of orbital angular momentum between the diquark and the charmed quark, and states with the unit of orbital angular momentum between the two light quarks. The orbital wave function of the former (latter) is symmetric (antisymmetric) under the exchange of two light quarks. To see this, one can define two independent relative momenta $\mathbf{k} = \frac{1}{2}(\mathbf{p}_1-\mathbf{p}_2)$ and $\mathbf{K} = \frac{1}{2}(\mathbf{p}_1+\mathbf{p}_2-2\mathbf{p}_c)$ from the two light quark momenta $\mathbf{p}_1$, $\mathbf{p}_2$ and the heavy quark momentum $\mathbf{p}_c$. (In the heavy quark limit, $\mathbf{p}_c$ can be set to zero.) Denoting the quantum numbers L_k and L_K as the eigenvalues of $\mathbf{L}_k^2$ and $\mathbf{L}_K^2$, the k-orbital momentum L_k describes relative orbital excitations of the two light quarks, and the K-orbital momentum L_K describes orbital excitations of the center of the mass of the two light quarks relative to the heavy quark [1]. The p-wave heavy baryon can be either in the $(L_k = 0, L_K = 1)$ K-state or the $(L_k = 1, L_K = 0)$ k-state. It is obvious that the orbital K-state (k-state) is symmetric (antisymmetric) under the interchange of $\mathbf{p}_1$ and $\mathbf{p}_2$.

The observed mass spectra and decay widths of charmed baryons are summarized in Table II (see also Fig. 1). B factories have provided a very rich source of charmed baryons both from B decays and from the continuum $e^+e^- \to c\bar{c}$. For example, several new excited charmed baryon states such

TABLE I: The p-wave charmed baryons and their quantum numbers, where S_ℓ (J_ℓ) is the total spin (angular momentum) of the two light quarks. The quantum number in the subscript labels J_ℓ. The quantum number in parentheses is referred to the spin of the baryon. In the quark model, the upper (lower) four multiplets have even (odd) orbital wave functions under the permutation of the two light quarks. That is, L_ℓ for the former is referred to the orbital angular momentum between the diquark and the charmed quark, while L_ℓ for the latter is the orbital angular momentum between the two light quarks. The explicit quark model wave functions for p-wave charmed baryons can be found in [4].

State	SU(3)	S_ℓ	L_ℓ	$J_\ell^{P_\ell}$	State	SU(3)	S_ℓ	L_ℓ	$J_\ell^{P_\ell}$
$\Lambda_{c1}(\frac{1}{2},\frac{3}{2})$	$\bar{\mathbf{3}}$	0	1	1^-	$\Xi_{c1}(\frac{1}{2},\frac{3}{2})$	$\bar{\mathbf{3}}$	0	1	1^-
$\Sigma_{c0}(\frac{1}{2})$	$\mathbf{6}$	1	1	0^-	$\Xi'_{c0}(\frac{1}{2})$	$\mathbf{6}$	1	1	0^-
$\Sigma_{c1}(\frac{1}{2},\frac{3}{2})$	$\mathbf{6}$	1	1	1^-	$\Xi'_{c1}(\frac{1}{2},\frac{3}{2})$	$\mathbf{6}$	1	1	1^-
$\Sigma_{c2}(\frac{3}{2},\frac{5}{2})$	$\mathbf{6}$	1	1	2^-	$\Xi'_{c2}(\frac{3}{2},\frac{5}{2})$	$\mathbf{6}$	1	1	2^-
$\tilde{\Sigma}_{c1}(\frac{1}{2},\frac{3}{2})$	$\mathbf{6}$	0	1	1^-	$\tilde{\Xi}'_{c1}(\frac{1}{2},\frac{3}{2})$	$\mathbf{6}$	0	1	1^-
$\tilde{\Lambda}_{c0}(\frac{1}{2})$	$\bar{\mathbf{3}}$	1	1	0^-	$\tilde{\Xi}_{c0}(\frac{1}{2})$	$\bar{\mathbf{3}}$	1	1	0^-
$\tilde{\Lambda}_{c1}(\frac{1}{2},\frac{3}{2})$	$\bar{\mathbf{3}}$	1	1	1^-	$\tilde{\Xi}_{c1}(\frac{1}{2},\frac{3}{2})$	$\bar{\mathbf{3}}$	1	1	1^-
$\tilde{\Lambda}_{c2}(\frac{3}{2},\frac{5}{2})$	$\bar{\mathbf{3}}$	1	1	2^-	$\tilde{\Xi}_{c2}(\frac{3}{2},\frac{5}{2})$	$\bar{\mathbf{3}}$	1	1	2^-

as $\Lambda_c(2765)^+, \Lambda_c(2880)^+, \Lambda_c(2940)^+$, $\Xi_c(2815), \Xi_c(2980)$ and $\Xi_c(3077)$ have been measured recently and they are not still not in the list of 2006 Particle Data Group [3]. By now, the $J^P = \frac{1}{2}^+$ and $\frac{1}{2}^-$ $\bar{\mathbf{3}}$ states: (Λ_c^+, Ξ_c^+,Ξ_c^0), ($\Lambda_c(2593)^+$, $\Xi_c(2790)^+,\Xi_c(2790)^0$), and $J^P = \frac{1}{2}^+$ and $\frac{3}{2}^+$ $\mathbf{6}$ states: $(\Omega_c, \Sigma_c, \Xi'_c)$, $(\Omega_c^*, \Sigma_c^*, \Xi_c'^*)$ are established. Notice that except for the parity of the lightest Λ_c^+, none of the other J^P quantum numbers given in Table II has been measured. One has to rely on the quark model to determine the J^P assignments.

In the following we discuss some of the new excited charmed baryon states:

- The highest $\Lambda_c(2940)^+$ was first discovered by BaBar in the D^0p decay mode [5] and confirmed by Belle in the decays $\Sigma_c^0\pi^+, \Sigma_c^{++}\pi^-$ which subsequently decay into $\Lambda_c^+\pi^+\pi^-$ [6, 7]. The state $\Lambda_c(2880)^+$ first observed by CLEO [8] in $\Lambda_c^+\pi^+\pi^-$ was also seen by BaBar in the D^0p spectrum [5]. It was originally conjectured that, based on its narrow width, $\Lambda_c(2880)^+$ might be a $\tilde{\Lambda}_{c0}^+(\frac{1}{2})$ state [8]. Recently, Belle has studied the experimental constraint on the J^P quantum numbers of $\Lambda_c(2880)^+$ [6]. The angular analysis of $\Lambda_c(2880)^+ \to \Sigma_c^{0,++}\pi^\pm$ indicates that $J = 5/2$ is favored over $J = 1/2$ or $3/2$, while the study of the resonant structure of $\Lambda_c(2880)^+ \to \Lambda_c^+\pi^+\pi^-$ implies the existence of the $\Sigma_c^*\pi$ intermediate states and $\Gamma(\Sigma_c^*\pi^\pm)/\Gamma(\Sigma_c\pi^\pm) = (24.1 \pm 6.4^{+1.1}_{-4.5})\%$. This value is in agreement with heavy quark symmetry predictions [9] and favors the $5/2^+$ over the $5/2^-$ assignment.[1] Therefore, it is not a $\tilde{\Lambda}_{c2}^+(\frac{5}{2})$ state either. Since $J_\ell = 2, S_\ell = 0, L = 2$ for the diquark system of $\Lambda_c(2880)^+$, this

[1] Strictly speaking, the argument in favor of the $5/2^+$ assignment is reached in [6] by considering only the F-wave contribution and neglecting the P-wave contribution to $\Lambda_c(2880)^+ \to \Sigma_c^*\pi$ (see [10] for more discussions).

TABLE II: Mass spectra and decay widths (in units of MeV) of charmed baryons. Experimental values are taken from the Particle Data Group [3] except $\Lambda_c(2880)$, $\Lambda_c(2940)$, $\Xi_c(2980)^{+,0}$, $\Xi_c(3077)^{+,0}$ and $\Omega_c(2768)$ for which we use the most recent available BaBar and Belle measurements.

State	quark content	J^P	Mass	Width
Λ_c^+	udc	$\frac{1}{2}^+$	2286.46 ± 0.14	
$\Lambda_c(2593)^+$	udc	$\frac{1}{2}^-$	2595.4 ± 0.6	$3.6^{+2.0}_{-1.3}$
$\Lambda_c(2625)^+$	udc	$\frac{3}{2}^-$	2628.1 ± 0.6	< 1.9
$\Lambda_c(2765)^+$	udc	$?^?$	2766.6 ± 2.4	50
$\Lambda_c(2880)^+$	udc	$\frac{5}{2}^+$	2881.5 ± 0.3	5.5 ± 0.6
$\Lambda_c(2940)^+$	udc	$?^?$	2938.8 ± 1.1	13.0 ± 5.0
$\Sigma_c(2455)^{++}$	uuc	$\frac{1}{2}^+$	2454.02 ± 0.18	2.23 ± 0.30
$\Sigma_c(2455)^+$	udc	$\frac{1}{2}^+$	2452.9 ± 0.4	< 4.6
$\Sigma_c(2455)^0$	ddc	$\frac{1}{2}^+$	2453.76 ± 0.18	2.2 ± 0.4
$\Sigma_c(2520)^{++}$	uuc	$\frac{3}{2}^+$	2518.4 ± 0.6	14.9 ± 1.9
$\Sigma_c(2520)^+$	udc	$\frac{3}{2}^+$	2517.5 ± 2.3	< 17
$\Sigma_c(2520)^0$	ddc	$\frac{3}{2}^+$	2518.0 ± 0.5	16.1 ± 2.1
$\Sigma_c(2800)^{++}$	uuc	$?^?$	2801^{+4}_{-6}	75^{+22}_{-17}
$\Sigma_c(2800)^+$	udc	$?^?$	2792^{+14}_{-5}	62^{+60}_{-40}
$\Sigma_c(2800)^0$	ddc	$?^?$	2802^{+4}_{-7}	61^{+28}_{-18}
Ξ_c^+	usc	$\frac{1}{2}^+$	2467.9 ± 0.4	
Ξ_c^0	dsc	$\frac{1}{2}^+$	2471.0 ± 0.4	
$\Xi_c^{\prime +}$	usc	$\frac{1}{2}^+$	2575.7 ± 3.1	
$\Xi_c^{\prime 0}$	dsc	$\frac{1}{2}^+$	2578.0 ± 2.9	
$\Xi_c(2645)^+$	usc	$\frac{3}{2}^+$	2646.6 ± 1.4	< 3.1
$\Xi_c(2645)^0$	dsc	$\frac{3}{2}^+$	2646.1 ± 1.2	< 5.5
$\Xi_c(2790)^+$	usc	$\frac{1}{2}^-$	2789.2 ± 3.2	< 15
$\Xi_c(2790)^0$	dsc	$\frac{1}{2}^-$	2791.9 ± 3.3	< 12
$\Xi_c(2815)^+$	usc	$\frac{3}{2}^-$	2816.5 ± 1.2	< 3.5
$\Xi_c(2815)^0$	dsc	$\frac{3}{2}^-$	2818.2 ± 2.1	< 6.5
$\Xi_c(2980)^+$	usc	$?^?$	2971.1 ± 1.7	25.2 ± 3.0
$\Xi_c(2980)^0$	dsc	$?^?$	2977.1 ± 9.5	43.5
$\Xi_c(3077)^+$	usc	$?^?$	3076.5 ± 0.6	6.2 ± 1.1
$\Xi_c(3077)^0$	dsc	$?^?$	3082.8 ± 2.3	5.2 ± 3.6
Ω_c^0	ssc	$\frac{1}{2}^+$	2697.5 ± 2.6	
$\Omega_c(2768)^0$	ssc	$\frac{3}{2}^+$	2768.3 ± 3.0	

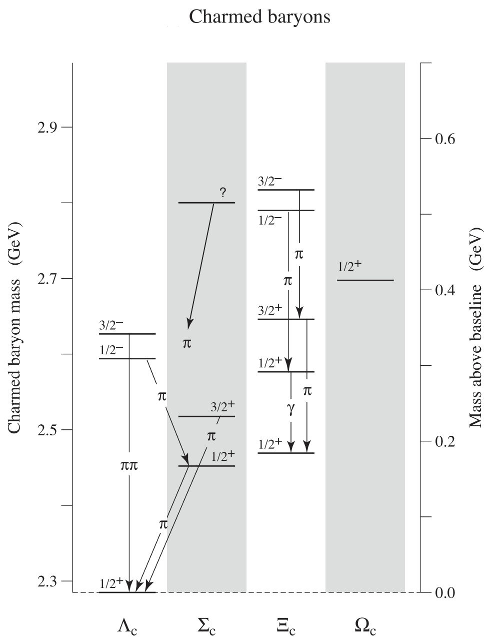

FIG. 1: Charmed baryons and some of their orbital excitations [3].

is the first observation of a d-wave charmed baryon. It is interesting to notice that, based on the diquark idea, the assignment $J^P = 5/2^+$ has already been predicted in [11] for the state $\Lambda_c(2880)$ before the Belle experiment. As for $\Lambda_c(2980)^+$, it was recently argued that it is an exotic molecular state of D^{*0} and p [12].

- The new charmed strange baryons $\Xi_c(2980)^+$ and $\Xi_c(3077)^+$ that decay into $\Lambda_c^+ K^- \pi^+$ were first observed by Belle [13] and confirmed by BaBar [14]. In the recent BaBar measurement [14], the $\Xi_c(2980)^+$ is found to decay resonantly through the intermediate state $\Sigma_c(2455)^{++} K^-$ with $4.9\,\sigma$ significance and non-resonantly to $\Lambda_c^+ K^- \pi^+$ with $4.1\,\sigma$ significance. With $5.8\,\sigma$ significance, the $\Xi_c(3077)^+$ is found to decay resonantly through $\Sigma_c(2455)^{++} K^-$, and with $4.6\,\sigma$ significance, it is found to decay through $\Sigma_c(2520)^{++} K^-$. The significance of the signal for the non-resonant decay $\Xi_c(3077)^+ \to \Lambda_c^+ K^- \pi^+$ is $1.4\,\sigma$.

- The highest isotriplet charmed baryons $\Sigma_c(2800)^{++,+,0}$ decaying into $\Lambda_c^+ \pi$ were first measured by Belle [15]. They are most likely to be the $J^P = 3/2^-$ Σ_{c2} states because the $\Sigma_{c2}(\frac{3}{2})$ baryon decays principally into the $\Lambda_c \pi$ system in a D-wave, while $\Sigma_{c1}(\frac{3}{2})$ decays mainly into the two pion system $\Lambda_c \pi\pi$. The state $\Sigma_{c0}(\frac{1}{2})$ can decay into $\Lambda_c \pi$ in an S-wave, but it is very broad with width of order 406 MeV [10]. Experimentally, it will be very difficult to observe it.

- The new $3/2^+$ $\Omega_c(2768)$ was recently observed by BaBar in the electromagnetic decay $\Omega_c(2768) \to \Omega_c \gamma$ [16]. With this new observation, the $3/2^+$ sextet is finally completed.

- Evidence of double charm states has been reported by SELEX in $\Xi_{cc}(3519)^+ \to \Lambda_c^+ K^- \pi^+$ [17]. Further observations of $\Xi_{cc}^{++} \to \Lambda_c^+ K^- \pi^+ \pi^+$ and $\Xi_{cc}^+ \to p D^+ K^-$ were also announced by SELEX [18]. However, none of the double charm states discovered by SELEX has been confirmed by FOCUS, BaBar [19] and Belle [7] despite the 10^6 Λ_c events produced in B factories versus 1630 Λ_c events observed at SELEX.

Charmed baryon spectroscopy has been studied extensively in various models. The interested readers are referred to [20] for further references. In heavy quark effective theory, the mass splittings between spin-$\frac{3}{2}$ and spin-$\frac{1}{2}$ sextet charmed baryon multiplets are governed by the chromomagnetic interactions so that

$$m_{\Sigma_c^*} - m_{\Sigma_c} = m_{\Xi_c'^*} - m_{\Xi_c'} = m_{\Omega_c^*} - m_{\Omega_c}, \tag{3.1}$$

up to corrections of $1/m_c$. This relation is borne out by experiment: $m_{\Sigma_c^{*+}} - m_{\Sigma_c^+} = 64.6 \pm 2.3$ MeV, $m_{\Xi_c'^{*+}} - m_{\Xi_c'^+} = 70.9 \pm 3.4$ MeV and $m_{\Omega_c^*} - m_{\Omega_c} = 70.8 \pm 1.5$ MeV.

IV. STRONG DECAYS

Due to the rich mass spectrum and the relatively narrow widths of the excited states, the charmed baryon system offers an excellent ground for testing the ideas and predictions of heavy quark symmetry and light flavor SU(3) symmetry. The pseudoscalar mesons involved in the strong decays of charmed baryons such as $\Sigma_c \to \Lambda_c\pi$ are soft. Therefore, heavy quark symmetry of the heavy quark and chiral symmetry of the light quarks will have interesting implications for the low-energy dynamics of heavy baryons interacting with the Goldstone bosons.

The strong decays of charmed baryons are most conveniently described by the heavy hadron chiral Lagrangians in which heavy quark symmetry and chiral symmetry are incorporated [21, 22]. The Lagrangian involves two coupling constants g_1 and g_2 for P-wave transitions between s-wave and s-wave baryons [21], six couplings h_2-h_7 for the S-wave transitions between s-wave and p-wave baryons, and eight couplings h_8-h_{15} for the D-wave transitions between s-wave and p-wave baryons [4]. The general chiral Lagrangian for heavy baryons coupling to the pseudoscalar mesons can be expressed compactly in terms of superfields. We will not write down the relevant Lagrangians here; instead the reader is referred to Eqs. (3.1) and (3.3) of [4]. Nevertheless, we list some of the partial widths derived from the Lagrangian [4]:

$$\begin{aligned}
\Gamma(\Sigma_c^* \to \Sigma_c\pi) &= \frac{g_1^2}{2\pi f_\pi^2}\frac{m_{\Sigma_c}}{m_{\Sigma_c^*}}p_\pi^3, \quad & \Gamma(\Sigma_c \to \Lambda_c\pi) &= \frac{g_2^2}{2\pi f_\pi^2}\frac{m_{\Lambda_c}}{m_{\Sigma_c}}p_\pi^3, \\
\Gamma(\Lambda_{c1}(1/2) \to \Sigma_c\pi) &= \frac{h_2^2}{2\pi f_\pi^2}\frac{m_{\Sigma_c}}{m_{\Lambda_{c1}}}E_\pi^2 p_\pi, \quad & \Gamma(\Sigma_{c0}(1/2) \to \Lambda_c\pi) &= \frac{h_3^2}{2\pi f_\pi^2}\frac{m_{\Lambda_c}}{m_{\Sigma_{c0}}}E_\pi^2 p_\pi, \\
\Gamma(\Sigma_{c1}(1/2) \to \Sigma_c\pi) &= \frac{h_4^2}{4\pi f_\pi^2}\frac{m_{\Sigma_c}}{m_{\Sigma_{c1}}}E_\pi^2 p_\pi, \quad & \Gamma(\tilde\Sigma_{c1}(1/2) \to \Sigma_c\pi) &= \frac{h_5^2}{4\pi f_\pi^2}\frac{m_{\Sigma_c}}{m_{\tilde\Sigma_{c1}}}E_\pi^2 p_\pi, \\
\Gamma(\tilde\Xi_{c0}(1/2) \to \Xi_c\pi) &= \frac{h_6^2}{2\pi f_\pi^2}\frac{m_{\Xi_c}}{m_{\tilde\Xi_{c0}}}E_\pi^2 p_\pi, \quad & \Gamma(\tilde\Lambda_{c1}(1/2) \to \Sigma_c\pi) &= \frac{h_7^2}{2\pi f_\pi^2}\frac{m_{\Sigma_c}}{m_{\tilde\Lambda_{c1}}}E_\pi^2 p_\pi,
\end{aligned} \tag{4.1}$$

where p_π is the pion's momentum and $f_\pi = 132$ MeV. Unfortunately, the decay $\Sigma_c^* \to \Sigma_c\pi$ is kinematically prohibited since the mass difference between Σ_c^* and Σ_c is only of order 65 MeV. Consequently, the coupling g_1 cannot be extracted directly from the strong decays of heavy baryons.

A. Strong decays of s-wave charmed baryons

In the framework of heavy hadron chiral pertrubation theory (HHChPT), one can use some measurements as input to fix the coupling g_2 which, in turn, can be used to predict the rates of other strong decays. We shall use $\Sigma_c \to \Lambda_c \pi$ as input [3]

$$\Gamma(\Sigma_c^{++}) = \Gamma(\Sigma_c^{++} \to \Lambda_c^+ \pi^+) \;=\; 2.23 \pm 0.30\,\mathrm{MeV}. \tag{4.2}$$

¿From which we obtain

$$|g_2| = 0.605^{+0.039}_{-0.043}\,, \tag{4.3}$$

where we have neglected the tiny contributions from electromagnetic decays. Note that $|g_2|$ obtained from $\Sigma_c^0 \to \Lambda_c^+ \pi^-$ has the same central value as Eq. (4.3) except that the errors are slightly large. If $\Sigma_c^* \to \Lambda_c \pi$ decays are employed as input, we will obtain $|g_2| = 0.57 \pm 0.04$ from $\Sigma_c^{*++} \to \Lambda_c^+ \pi^+$ and 0.60 ± 0.04 from $\Sigma_c^{*0} \to \Lambda_c^+ \pi^-$. Hence, it is preferable to use the measurement of $\Sigma_c^{++} \to \Lambda_c^+ \pi^+$ to fix $|g_2|$.[2]

As pointed out in [21], within in the framework of the non-relativistic quark model, the couplings g_1 and g_2 can be related to g_A^q, the axial-vector coupling in a single quark transition of $u \to d$, via

$$g_1 = \frac{4}{3} g_A^q, \qquad\qquad g_2 = \sqrt{\frac{2}{3}} g_A^q. \tag{4.4}$$

Using $g_A^q = 0.75$ which is required to reproduce the correct value of $g_A^N = 1.25$, we obtain

$$g_1 = 1, \qquad\qquad g_2 = 0.61\,. \tag{4.5}$$

Hence, the quark model prediction is in good agreement with experiment, but deviates 2σ from the large-N_c argument: $|g_2| = g_A^N/\sqrt{2} = 0.88$ [24]. Applying (4.3) leads to (see also Table III)

$$\begin{aligned}
\Gamma(\Xi_c^{\prime *+}) = \Gamma(\Xi_c^{\prime *+} \to \Xi_c^+ \pi^0, \Xi_c^0 \pi^+) \;&=\; \frac{g_2^2}{4\pi f_\pi^2}\left(\frac{1}{2}\frac{m_{\Xi_c^+}}{m_{\Xi_c^{\prime +}}} p_\pi^3 + \frac{m_{\Xi_c^0}}{m_{\Xi_c^{\prime +}}} p_\pi^3\right) = (2.8 \pm 0.4)\,\mathrm{MeV}, \\
\Gamma(\Xi_c^{\prime *0}) = \Gamma(\Xi_c^{\prime *0} \to \Xi_c^+ \pi^-, \Xi_c^0 \pi^0) \;&=\; \frac{g_2^2}{4\pi f_\pi^2}\left(\frac{m_{\Xi_c^+}}{m_{\Xi_c^{\prime 0}}} p_\pi^3 + \frac{1}{2}\frac{m_{\Xi_c^0}}{m_{\Xi_c^{\prime 0}}} p_\pi^3\right) = (2.9 \pm 0.4)\,\mathrm{MeV}.
\end{aligned} \tag{4.6}$$

Note that we have neglected the effect of $\Xi_c - \Xi_c'$ mixing in calculations (for recent considerations, see [29, 30]). Therefore, the predicted total width of $\Xi_c^{\prime *+}$ is in the vicinity of the current limit $\Gamma(\Xi_c^{\prime *+}) < 3.1$ MeV [31].

It is clear from Table III that the predicted widths of Σ_c^{++} and Σ_c^0 by HHChPT are in good agreement with experiment. The strong decay width of Σ_c is smaller than that of Σ_c^* by a factor of ~ 7, although they will become the same in the limit of heavy quark symmetry. This is ascribed to the fact that the pion's momentum is around 90 MeV in the decay $\Sigma_c \to \Lambda_c \pi$ while it is two times bigger in $\Sigma_c^* \to \Lambda_c \pi$. Since Σ_c states are significantly narrower than their spin-3/2 counterparts, this explains why the measurement of their widths came out much later. Instead of using the data to fix the coupling constants in a model-independent manner, there exist some calculations of couplings in various models such as the relativistic light-front model [25], the relativistic three-quark model [26] and light-cone sum rules [27, 32]. The results are summarized in Table III.

It is worth remarking that although the coupling g_1 cannot be determined directly from the strong decay such as $\Sigma_c^* \to \Sigma_c \pi$, some information of g_1 can be learned from the radiative decay $\Xi_c^{\prime *0} \to \Xi_c^0 \gamma$, which is prohibited at tree level by SU(3) symmetry but can be induced by chiral loops. A measurement of $\Gamma(\Xi_c^{\prime *0} \to \Xi_c^0 \gamma)$ will yield two possible solutions for g_1. Assuming the validity of the quark model relations among different coupling constants, the experimental value of g_2 implies $|g_1| = 0.93 \pm 0.16$ [23] (see also Sec. VIII.A).

[2] For previous efforts of extracting g_2 from experiment using HHChPT, see [4, 23].

TABLE III: Decay widths (in units of MeV) of s-wave charmed baryons. Theoretical predictions of [25] are taken from Table IV of [26].

Decay	Expt. [3]	HHChPT [10]	Tawfiq et al. [25]	Ivanov et al. [26]	Huang et al. [27]	Albertus et al. [28]
$\Sigma_c^{++} \to \Lambda_c^+\pi^+$	2.23 ± 0.30	input	1.51 ± 0.17	2.85 ± 0.19	2.5	2.41 ± 0.07
$\Sigma_c^{+} \to \Lambda_c^+\pi^0$	< 4.6	2.6 ± 0.4	1.56 ± 0.17	3.63 ± 0.27	3.2	2.79 ± 0.08
$\Sigma_c^{0} \to \Lambda_c^+\pi^-$	2.2 ± 0.4	2.2 ± 0.3	1.44 ± 0.16	2.65 ± 0.19	2.4	2.37 ± 0.07
$\Sigma_c(2520)^{++} \to \Lambda_c^+\pi^+$	14.9 ± 1.9	16.7 ± 2.3	11.77 ± 1.27	21.99 ± 0.87	8.2	17.52 ± 0.75
$\Sigma_c(2520)^{+} \to \Lambda_c^+\pi^0$	< 17	17.4 ± 2.3			8.6	17.31 ± 0.74
$\Sigma_c(2520)^{0} \to \Lambda_c^+\pi^-$	16.1 ± 2.1	16.6 ± 2.2	11.37 ± 1.22	21.21 ± 0.81	8.2	16.90 ± 0.72
$\Xi_c(2645)^{+} \to \Xi_c^{0,+}\pi^{+,0}$	< 3.1	2.8 ± 0.4	1.76 ± 0.14	3.04 ± 0.37		3.18 ± 0.10
$\Xi_c(2645)^{0} \to \Xi_c^{+,0}\pi^{-,0}$	< 5.5	2.9 ± 0.4	1.83 ± 0.06	3.12 ± 0.33		3.03 ± 0.10

B. Strong decays of p-wave charmed baryons

Some of the S-wave and D-wave couplings of p-wave baryons to s-wave baryons can be determined. In principle, the coupling h_2 is readily extracted from $\Lambda_c(2593)^+ \to \Sigma_c^0\pi^+$ with $\Lambda_c(2593)^+$ identified as $\Lambda_{c1}(\frac{1}{2})^+$. However, since $\Lambda_c(2593)^+ \to \Sigma_c\pi$ is kinematically barely allowed, the finite width effects of the intermediate resonant states will become important [33].

Pole contributions to the decays $\Lambda_c(2593)^+, \Lambda_c(2625)^+ \to \Lambda_c^+\pi\pi$ have been considered in [4, 27, 34] with the finite width effects included. The intermediate states of interest are Σ_c and Σ_c^* poles. The resonant contribution arises from the Σ_c pole, while the non-resonant term receives a contribution from the Σ_c^* pole. (Since $\Lambda_c(2593)^+, \Lambda_c(2625)^+ \to \Lambda_c^*\pi$ are not kinematically allowed, the Σ_c^* pole is not a resonant contribution.) The decay rates thus depend on two coupling constants h_2 and h_8. The decay rate for the process $\Lambda_{c_1}^+(2593) \to \Lambda_c^+\pi^+\pi^-$ can be calculated in the framework of heavy hadron chiral perturbation theory to be [10]

$$\begin{aligned} \Gamma(\Lambda_c(2593) \to \Lambda_c^+\pi\pi) &= 14.48h_2^2 + 27.54h_8^2 - 3.11h_2h_8, \\ \Gamma(\Lambda_c(2625) \to \Lambda_c^+\pi\pi) &= 0.648h_2^2 + 0.143 \times 10^6 h_8^2 - 28.6h_2h_8. \end{aligned} \tag{4.7}$$

It is clear that the limit on $\Gamma(\Lambda_c(2625))$ gives an upper bound on h_8 of order 10^{-3} (in units of MeV^{-1}), whereas the decay width of $\Lambda_c(2593)$ is entirely governed by the coupling h_2. This indicates that the direct non-resonant $\Lambda_c^+\pi\pi$ cannot be described by the Σ_c^* pole alone. Identifying the calculated $\Gamma(\Lambda_c(2593) \to \Lambda_c^+\pi\pi)$ with the resonant one, we find

$$|h_2| = 0.427^{+0.111}_{-0.100}\,, \qquad |h_8| \le 3.57 \times 10^{-3}\,. \tag{4.8}$$

Assuming that the total decay width of $\Lambda_c(2593)$ is saturated by the resonant $\Lambda_c^+\pi\pi$ 3-body decays, Pirjol and Yan obtained $|h_2| = 0.572^{+0.322}_{-0.197}$ and $|h_8| \le (3.50 - 3.68) \times 10^{-3}\,\mathrm{MeV}^{-1}$ [4]. Using the updated hadron masses and $\Gamma(\Lambda_c(2593) \to \Lambda_c^+\pi\pi)$,[3] we find $|h_2| = 0.499^{+0.134}_{-0.100}$. Taking

[3] The CLEO result $\Gamma(\Lambda_c(2593)) = 3.9^{+2.4}_{-1.6}$ MeV [35] is used in [4] to fix h_2.

TABLE IV: Same as Table III except for p-wave charmed baryons [10].

Decay	Expt. [3]	HHChPT [10]	Tawfiq et al. [25]	Ivanov et al. [26]	Huang et al. [27]	Zhu [32]
$\Lambda_c(2593)^+ \to (\Lambda_c^+\pi\pi)_R$	$2.63^{+1.56}_{-1.09}$	input			2.5	
$\Lambda_c(2593)^+ \to \Sigma_c^{++}\pi^-$	$0.65^{+0.41}_{-0.31}$	$0.62^{+0.37}_{-0.26}$	1.47 ± 0.57	0.79 ± 0.09	$0.55^{+1.3}_{-0.55}$	0.64
$\Lambda_c(2593)^+ \to \Sigma_c^0\pi^+$	$0.67^{+0.41}_{-0.31}$	$0.67^{+0.40}_{-0.28}$	1.78 ± 0.70	0.83 ± 0.09	0.89 ± 0.86	0.86
$\Lambda_c(2593)^+ \to \Sigma_c^+\pi^0$		$1.34^{+0.79}_{-0.55}$	1.18 ± 0.46	0.98 ± 0.12	1.7 ± 0.49	1.2
$\Lambda_c(2625)^+ \to \Sigma_c^{++}\pi^-$	< 0.10	≤ 0.028	0.44 ± 0.23	0.076 ± 0.009	0.013	0.011
$\Lambda_c(2625)^+ \to \Sigma_c^0\pi^+$	< 0.09	≤ 0.028	0.47 ± 0.25	0.080 ± 0.009	0.013	0.011
$\Lambda_c(2625)^+ \to \Sigma_c^+\pi^0$		≤ 0.040	0.42 ± 0.22	0.095 ± 0.012	0.013	0.011
$\Lambda_c(2625)^+ \to \Lambda_c^+\pi\pi$	< 1.9	≤ 0.21			0.11	
$\Sigma_c(2800)^{++} \to \Lambda_c\pi, \Sigma_c^{(*)}\pi$	75^{+22}_{-17}	input				
$\Sigma_c(2800)^{+} \to \Lambda_c\pi, \Sigma_c^{(*)}\pi$	62^{+60}_{-40}	input				
$\Sigma_c(2800)^{0} \to \Lambda_c\pi, \Sigma_c^{(*)}\pi$	61^{+28}_{-18}	input				
$\Xi_c(2790)^+ \to \Xi_c'^{0,+}\pi^{+,0}$	< 15	$7.7^{+4.5}_{-3.2}$				
$\Xi_c(2790)^0 \to \Xi_c'^{+,0}\pi^{-,0}$	< 12	$8.1^{+4.8}_{-3.4}$				
$\Xi_c(2815)^+ \to \Xi_c^{*+,0}\pi^{0,+}$	< 3.5	$3.2^{+1.9}_{-1.3}$	2.35 ± 0.93	0.70 ± 0.04		
$\Xi_c(2815)^0 \to \Xi_c^{*+,0}\pi^{-,0}$	< 6.5	$3.5^{+2.0}_{-1.4}$				

into account the fact that the Σ_c and Σ_c^* poles only describe the resonant contributions to the total width of $\Lambda_c(2593)$, we finally reach the h_2 value given in (4.8). Taking into account the threshold (or finite width) effect in the strong decay $\Lambda_c(2593)^+ \to \Lambda_c\pi\pi$, a slightly small coupling $h_2^2 = 0.24^{+0.23}_{-0.11}$ is obtained in [33]. For the spin-$\frac{3}{2}$ state $\Lambda_c(2625)$, its decay is dominated by the three-body channel $\Lambda_c^+\pi\pi$ as the major two-body decay $\Sigma_c\pi$ is a D-wave one.

Some information on the coupling h_{10} cane be inferred from the strong decays of $\Lambda_c(2800)$. As noticed in passing, the states $\Sigma_c(2800)^{++,+,0}$ are most likely to be $\Sigma_{c2}(\frac{3}{2})$. Assuming their widths are dominated by the two-body modes $\Lambda_c\pi$, $\Sigma_c\pi$ and $\Lambda_c^*\pi$, we have [4]

$$\begin{aligned}\Gamma\left(\Sigma_{c2}(\frac{3}{2})^{++}\right) &\approx \Gamma\left(\Sigma_{c2}(\frac{3}{2})^{++} \to \Lambda_c^+\pi^+\right) + \Gamma\left(\Sigma_{c2}(\frac{3}{2})^{++} \to \Sigma_c^+\pi^+\right) + \Gamma\left(\Sigma_{c2}(\frac{3}{2})^{++} \to \Sigma_c^{*+}\pi^+\right) \\ &= \frac{4h_{10}^2}{15\pi f_\pi^2}\frac{m_{\Lambda_c}}{m_{\Sigma_{c2}}}p_c^5 + \frac{h_{11}^2}{10\pi f_\pi^2}\frac{m_{\Sigma_c}}{m_{\Sigma_{c2}}}p_c^5 + \frac{h_{11}^2}{10\pi f_\pi^2}\frac{m_{\Sigma_c^*}}{m_{\Sigma_{c2}}}p_c^5, \end{aligned} \tag{4.9}$$

and similar expressions for $\Sigma_c(2800)^+$ and $\Sigma_c(2800)^0$. Using the quark model relation $h_{11}^2 = 2h_{10}^2$ [see also Eq. (4.12)] and the measured widths of $\Sigma_c(2800)^{++,+,0}$ (Table II), we obtain

$$|h_{10}| = (0.85^{+0.11}_{-0.08}) \times 10^{-3}\,\mathrm{MeV}^{-1}\,. \tag{4.10}$$

Since the state $\Lambda_{c1}(\frac{3}{2})$ is broader, even a small mixing of $\Lambda_{c2}(\frac{3}{2})$ with $\Lambda_{c1}(\frac{3}{2})$ could enhance the decay width of the former [4]. In this case, the above value for h_{10} should be regarded as an upper limit of $|h_{10}|$. Using the quark model relation $|h_8| = |h_{10}|$ (see Eq. (4.12) below), the calculated partial widths of $\Lambda_c(2625)^+$ are shown in Table IV.

The $\Xi_c(2790)$ and $\Xi_c(2815)$ baryons form a doublet $\Xi_{c1}(\frac{1}{2},\frac{3}{2})$. $\Xi_c(2790)$ decays to $\Xi_c'\pi$, while $\Xi_c(2815)$ decays to $\Xi_c\pi\pi$, resonating through Ξ_c^*, i.e. $\Xi_c(2645)$. Using the coupling h_2 obtained (4.8) and the experimental observation that the $\Xi_c\pi\pi$ mode in $\Xi_c(2815)$ decays is consistent with being entirely via $\Xi_c(2645)\pi$, the predicted $\Xi_c(2790)$ and $\Xi_c(2815)$ widths are shown in Table IV and they are consistent with the current experimental limits.

Couplings other than h_2 and h_{10} can be related to each other via the quark model. The S-wave couplings between the s-wave and the p-wave baryons are related by [4]

$$\frac{|h_3|}{|h_4|}=\frac{\sqrt{3}}{2},\quad \frac{|h_2|}{|h_4|}=\frac{1}{2},\quad \frac{|h_5|}{|h_6|}=\frac{2}{\sqrt{3}},\quad \frac{|h_5|}{|h_7|}=1\,. \tag{4.11}$$

The D-wave couplings satisfy the relations

$$|h_8|=|h_9|=|h_{10}|,\quad \frac{|h_{11}|}{|h_{10}|}=\frac{|h_{15}|}{|h_{14}|}=\sqrt{2},\quad \frac{|h_{12}|}{|h_{13}|}=2,\quad \frac{|h_{14}|}{|h_{13}|}=1\,. \tag{4.12}$$

The reader is referred to [4] for further details.

V. LIFETIMES

The lifetime differences among the charmed mesons D^+, D^0 and charmed baryons have been studied extensively both experimentally and theoretically since late 1970s. It was realized very early that the naive parton model gives the same lifetimes for all heavy particles containing a heavy quark Q and that the underlying mechanism for the decay width differences and the lifetime hierarchy of heavy hadrons comes mainly from the spectator effects like W-exchange and Pauli interference due to the identical quarks produced in the heavy quark decay and in the charmed baryons (for a review, see [2, 36, 37]). The spectator effects were expressed in 1980s in terms of local four-quark operators by relating the total widths to the imaginary part of certain forward scattering amplitudes [38–40]. (The spectator effects for charmed baryons were first studied in [41].) With the advent of heavy quark effective theory (HQET), it was recognized in early 1990s that nonperturbative corrections to the parton picture can be systematically expanded in powers of $1/m_Q$ [42, 43]. Subsequently, it was demonstrated that this $1/m_Q$ expansion is applicable not only to global quantities such as lifetimes, but also to local quantities, e.g. the lepton spectrum in the semileptonic decays of heavy hadrons [44]. Therefore, the above-mentioned phenomenological work in 1980s acquired a firm theoretical footing in 1990s, namely the heavy quark expansion (HQE), which is a generalization of the operator product expansion (OPE) in $1/m_Q$. Within this QCD-based framework, some phenomenological assumptions can be turned into some coherent and quantitative statements and nonperturbative effects can be systematically studied.

Based on the OPE approach for the analysis of inclusive weak decays, the inclusive rate of the charmed baryon is schematically represented by

$$\Gamma(\mathcal{B}_c\to f)=\frac{G_F^2 m_c^5}{192\pi^3}V_{\rm CKM}\left(A_0+\frac{A_2}{m_c^2}+\frac{A_3}{m_c^3}+\mathcal{O}(\frac{1}{m_c^4})\right). \tag{5.1}$$

The A_0 term comes from the c quark decay and is common to all charmed hadrons. There is no linear $1/m_Q$ corrections to the inclusive decay rate due to the lack of gauge-invariant dimension-four operators [42, 45], a consequence known as Luke's theorem [46]. Nonperturbative corrections start at order $1/m_Q^2$ and they are model independent. Spectator effects in inclusive decays due to the Pauli interference and W-exchange contributions account for $1/m_c^3$ corrections and they have two eminent features: First, the estimate of spectator effects is model dependent; the hadronic four-quark matrix elements are usually evaluated by assuming the factorization approximation for mesons and the quark model for baryons. Second, there is a two-body phase-space enhancement factor of $16\pi^2$ for spectator effects relative to the three-body phase space for heavy quark decay. This implies that spectator effects, being of order $1/m_c^3$, are comparable to and even exceed the $1/m_c^2$ terms.

The lifetimes of charmed baryons are measured to be [3]

$$\begin{aligned} \tau(\Lambda_c^+) &= (200 \pm 6) \times 10^{-15} s, \qquad & \tau(\Xi_c^+) &= (442 \pm 26) \times 10^{-15} s, \\ \tau(\Xi_c^0) &= (112^{+13}_{-10}) \times 10^{-15} s, \qquad & \tau(\Omega_c^0) &= (69 \pm 12) \times 10^{-15} s. \end{aligned} \tag{5.2}$$

As we shall see below, the lifetime hierarchy $\tau(\Xi_c^+) > \tau(\Lambda_c^+) > \tau(\Xi_c^0) > \tau(\Omega_c^0)$ is qualitatively understandable in the OPE approach but not quantitatively.

In general, the total width of the charmed baryon $\mathcal{B}_c$ receives contributions from inclusive nonleptonic and semileptonic decays: $\Gamma(\mathcal{B}_c) = \Gamma_{\rm NL}(\mathcal{B}_c) + \Gamma_{\rm SL}(\mathcal{B}_c)$. The nonleptonic contribution can be decomposed into

$$\Gamma_{\rm NL}(\mathcal{B}_c) = \Gamma^{\rm dec}(\mathcal{B}_c) + \Gamma^{\rm ann}(\mathcal{B}_c) + \Gamma^{\rm int}_-(\mathcal{B}_c) + \Gamma^{\rm int}_+(\mathcal{B}_c), \tag{5.3}$$

corresponding to the c-quark decay, the W-exchange contribution, destructive and constructive Pauli interferences. It is known that the inclusive decay rate is governed by the imaginary part of an effective nonlocal forward transition operator T. Therefore, $\Gamma^{\rm dec}$ corresponds to the imaginary part of Fig. 2(a) sandwiched between the same $\mathcal{B}_c$ states. At the Cabibbo-allowed level, $\Gamma^{\rm dec}$ represents the decay rate of $c \to su\bar{d}$, and $\Gamma^{\rm ann}$ denotes the contribution due to the W-exchange diagram $cd \to us$. The interference $\Gamma^{\rm int}_-$ ($\Gamma^{\rm int}_+$) arises from the destructive (constructive) interference between the u (s) quark produced in the c-quark decay and the spectator u (s) quark in the charmed baryon $\mathcal{B}_c$. Notice that the constructive Pauli interference is unique to the charmed baryon sector as it does not occur in the bottom baryon sector. From the quark content of the charmed baryons (see Table II), it is clear that at the Cabibbo-allowed level, the destructive interference occurs in Λ_c^+ and Ξ_c^+ decays, while Ξ_c^+, Ξ_c^0 and Ω_c^0 can have $\Gamma^{\rm int}_+$. Since Ω_c^0 contains two s quarks, it is natural to expect that $\Gamma^{\rm int}_+(\Omega_c^0) \gg \Gamma^{\rm int}_+(\Xi_c)$. W-exchange occurs only for Ξ_c^0 and Λ_c^+ at the same Cabibbo-allowed level. In the heavy quark expansion approach, the above-mentioned spectator effects can be described in terms of the matrix elements of local four-quark operators.

Within this QCD-based heavy quark expansion approach, some phenomenological assumptions can be turned into some coherent and quantitative statements and nonperturbative effects can be systematically studied. To begin with, we write down the general expressions for the inclusive decay widths of charmed hadrons. Under the heavy quark expansion, the inclusive nonleptonic decay rate of a charmed baryon $\mathcal{B}_c$ is given by [42, 43]

$$\begin{aligned} \Gamma_{\rm NL}(\mathcal{B}_c) &= \frac{G_F^2 m_c^5}{192\pi^3} N_c V_{\rm CKM} \frac{1}{2m_{\mathcal{B}_c}} \Bigg\{ \left(c_1^2 + c_2^2 + \frac{2c_1c_2}{N_c}\right) \Big[I_0(x,0,0) \langle \mathcal{B}_c|\bar{c}c|\mathcal{B}_c\rangle \\ &- \frac{1}{m_c^2} I_1(x,0,0) \langle \mathcal{B}_c|\bar{c}\sigma\cdot Gc|\mathcal{B}_c\rangle \Big] - \frac{4}{m_c^2}\frac{2c_1c_2}{N_c} I_2(x,0,0)\langle \mathcal{B}_c|\bar{c}\sigma\cdot Gc|\mathcal{B}_c\rangle \Bigg\} \\ &+ \frac{1}{2m_{\mathcal{B}_c}} \langle \mathcal{B}_c|\mathcal{L}_{\rm spec}|\mathcal{B}_c\rangle + \mathcal{O}\left(\frac{1}{m_c^4}\right), \end{aligned} \tag{5.4}$$

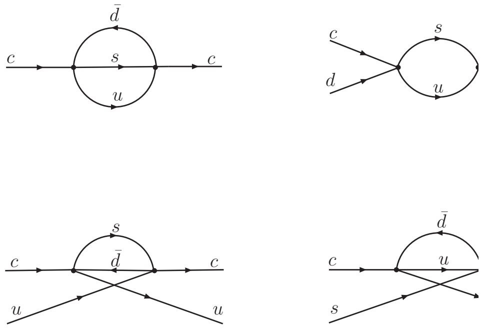

FIG. 2: Contributions to nonleptonic decay rates of charmed baryons from four-quark operators: (a) c-quark decay, (b) W-exchange, (c) destructive Pauli interference and (d) constructive interference.

where $\sigma\cdot G = \sigma_{\mu\nu}G^{\mu\nu}$, $x = (m_s/m_c)^2$, N_c is the number of colors, c_1, c_2 are Wilson coefficient functions, $N_c = 3$ is the number of color and $V_{\rm CKM}$ takes care of the relevant CKM matrix elements. In the above equation, $I_{0,1,2}$ are phase-space factors

$$\begin{aligned} I_0(x,0,0) &= (1-x^2)(1-8x+x^2) - 12x^2\ln x, \\ I_1(x,0,0) &= \frac{1}{2}(2-x\frac{d}{dx})I_0(x,0,0) = (1-x)^4, \\ I_2(x,0,0) &= (1-x)^3, \end{aligned} \tag{5.5}$$

for $c\to su\bar{d}$ transition.

In heavy quark effective theory, the two-body matrix element $\langle\mathcal{B}_c|\bar{c}c|\mathcal{B}_c\rangle$ in Eq. (5.4) can be recast to

$$\frac{\langle\mathcal{B}_c|\bar{c}c|\mathcal{B}_c\rangle}{2m_{\mathcal{B}_c}} = 1 - \frac{K_H}{2m_c^2} + \frac{G_H}{2m_c^2}, \tag{5.6}$$

with

$$\begin{aligned} K_H &\equiv -\frac{1}{2m_{\mathcal{B}_c}}\langle\mathcal{B}_c|\bar{c}(iD_\perp)^2c|\mathcal{B}_c\rangle = -\lambda_1, \\ G_H &\equiv \frac{1}{2m_{\mathcal{B}_c}}\langle\mathcal{B}_c|\bar{c}\frac{1}{2}\sigma\cdot Gc|\mathcal{B}_c\rangle = d_H\lambda_2, \end{aligned} \tag{5.7}$$

where $d_H = 0$ for the antitriplet baryon and $d_H = 4$ for the spin-$\frac{1}{2}$ sextet baryon. It should be stressed that the expression (5.6) is model independent and it contains nonperturbative kinetic and chromomagnetic effects which are usually absent in the quark model calculations. The nonperturbative HQET parameters λ_1 and λ_2 are independent of the heavy quark mass. Numerically, we shall use $\lambda_1^{\rm baryon} = -(0.4\pm0.2)\,{\rm GeV}^2$ [47] and $\lambda_2^{\rm baryon} = 0.055\,{\rm GeV}^2$ for charmed baryons [48]. Spectator effects in inclusive decays of charmed hadrons are described by the dimension-six four-quark operators $\mathcal{L}_{\rm spec}$ in Eq. (5.4) at order $1/m_c^3$. Its complete expression can be found in, for example, Eq. (2.4) of [48].

For inclusive semileptonic decays, there is an additional spectator effect in charmed-baryon semileptonic decay originating from the Pauli interference of the s quark for charmed baryons Ξ_c and Ω_c [49]. The general expression of the inclusive semileptonic widths is given by

$$\begin{aligned} \Gamma_{\rm SL}(\mathcal{B}_c) &= \frac{G_F^2m_c^5}{192\pi^3}V_{\rm CKM}\,\frac{\eta(x,x_\ell,0)}{2m_{\mathcal{B}_c}}\Big[I_0(x,0,0)\langle\mathcal{B}_c|\bar{c}c|\mathcal{B}_c\rangle - \frac{1}{m_c^2}\,I_1(x,0,0)\langle\mathcal{B}_c|\bar{c}\sigma\cdot Gc|\mathcal{B}_c\rangle\Big] \\ &- \frac{G_F^2m_c^2}{6\pi}|V_{cs}|^2\frac{1}{2m_{\mathcal{B}_c}}(1-x)^2\Big[(1+\frac{x}{2})(\bar{c}s)(\bar{s}c) - (1+2x)\bar{c}(1-\gamma_5)s\bar{s}(1+\gamma_5)c\Big], \end{aligned} \tag{5.8}$$

where $\eta(x, x_\ell, 0)$ with $x_\ell = (m_\ell/m_Q)^2$ is the QCD radiative correction to the semileptonic decay rate and its general analytic expression is given in [50]. Since both nonleptonic and semileptonic decay widths scale with the fifth power of the charmed quark mass, it is very important to fix the value of m_c. It is found that the experimental values for D^+ and D^0 semileptonic widths [3] can be fitted by the quark pole mass $m_c = 1.6$ GeV. Taking $m_s = 170$ MeV, we obtain the charmed-baryon semileptonic decay rates

$$\begin{aligned}
&\Gamma(\Lambda_c \to Xe\bar{\nu}) = \Gamma(\Xi_c \to Xe\bar{\nu}) = \ 1.533 \times 10^{-13}\text{GeV}, \\
&\Gamma(\Omega_c \to Xe\bar{\nu}) = \ 1.308 \times 10^{-13}\text{GeV}.
\end{aligned} \tag{5.9}$$

The prediction (5.9) for the Λ_c baryon is in good agreement with experiment [3]

$$\Gamma(\Lambda_c \to Xe\bar{\nu})_{\text{expt}} = \ (1.480 \pm 0.559) \times 10^{-13}\text{GeV}. \tag{5.10}$$

We shall see beolw that the Pauli interference effect in the semileptonic decays of Ξ_c and Ω_c can be very significant, in particular for the latter.

The baryon matrix element of the four-quark operator $\langle \mathcal{B}_c|(\bar{c}q_1)(\bar{q}_2q_3)|\mathcal{B}_c\rangle$ with $(\bar{q}_1q_2) = \bar{q}_1\gamma_\mu(1-\gamma_5)q_2$ is customarily evaluated using the quark model. In the non-relativistic quark model (for early related studies, see [38, 39]), the matrix element is governed by the charmed baryon wave function at origin, $|\psi^{\mathcal{B}_c}_{cq}(0)|^2$, which can be related to the charmed meson wave function $|\psi^D_{cq}(0)|^2$. For example, the hyperfine splittings between Σ_c^* and Σ_c, and between D^* and D separately yield [51]

$$|\psi^{\Lambda_c}_{cq}(0)|^2 = |\psi^{\Sigma_c}_{cq}(0)|^2 = \frac{4}{3}\,\frac{m_{\Sigma_c^*} - m_{\Sigma_c}}{m_{D^*} - m_D}\,|\psi^D_{c\bar{q}}(0)|^2. \tag{5.11}$$

This relation is supposed to be robust as $|\psi_{cq}(0)|^2$ determined in this manner does not depend on the strong coupling α_s and the light quark mass m_q directly. Defining

$$|\psi^{\mathcal{B}_c}_{cq}(0)|^2 = r_{\mathcal{B}_c}|\psi^D_{cq}(0)|^2, \tag{5.12}$$

we have

$$r_{\Lambda_c} = \frac{4}{3}\,\frac{m_{\Sigma_c^*} - m_{\Sigma_c}}{m_{D^*} - m_D}, \quad r_{\Xi_c} = \frac{4}{3}\,\frac{m_{\Xi_c^*} - m_{\Xi_c'}}{m_{D^*} - m_D}, \quad r_{\Omega_c} = \frac{4}{3}\,\frac{m_{\Omega_c^*} - m_{\Omega_c}}{m_{D^*} - m_D}. \tag{5.13}$$

In terms of the parameter $r_{\mathcal{B}_c}|\psi^D_{cq}(0)|^2$ we have [48]

$$\begin{aligned}
\Gamma^{\text{ann}}(\Lambda_c) &= \frac{G_F^2m_c^2}{\pi}\,r_{\Lambda_c}(1-x)^2\Big(\eta(c_1^2+c_2^2) - 2c_1c_2\Big)|\psi^D(0)|^2, \\
\Gamma^{\text{int}}_-(\Lambda_c) &= -\frac{G_F^2m_c^2}{4\pi}\,r_{\Lambda_c}(1-x)^2(1+x)\Big(\eta c_1^2 - 2c_1c_2 - N_cc_2^2\Big)|\psi^D(0)|^2, \\
\Gamma^{\text{ann}}(\Xi_c)/r_{\Xi_c} &= \Gamma^{\text{ann}}(\Lambda_c)/r_{\Lambda_c}, \qquad \Gamma^{\text{int}}_-(\Xi_c^+)/r_{\Xi_c} = \Gamma^{\text{int}}_-(\Lambda_c)/r_{\Lambda_c}, \\
\Gamma^{\text{int}}_+(\Xi_c) &= -\frac{G_F^2m_c^2}{4\pi}\,r_{\Xi_c}(1-x^2)(1+x)\Big(\eta c_2^2 - 2c_1c_2 - N_cc_1^2\Big)|\psi^D(0)|^2, \\
\Gamma^{\text{int}}_+(\Omega_c) &= -\frac{G_F^2m_c^2}{6\pi}\,r_{\Omega_c}(1-x^2)(5+x)\Big(\eta c_2^2 - 2c_1c_2 - N_cc_1^2\Big)|\psi^D(0)|^2, \\
\Gamma^{\text{ann}}(\Omega_c) &= 6\frac{G_F^2m_c^2}{\pi}\,r_{\Omega_c}(1-x^2)\Big(\eta(c_1^2+c_2^2) - 2c_1c_2\Big)|\psi^D(0)|^2, \\
\Gamma^{\text{int}}(\Xi_c \to Xe\bar{\nu}) &= \frac{G_F^2m_c^2}{4\pi}\,r_{\Xi_c}(1-x^2)(1+x)|\psi^D(0)|^2, \\
\Gamma^{\text{int}}(\Omega_c \to Xe\bar{\nu}) &= \frac{G_F^2m_c^2}{6\pi}\,r_{\Omega_c}(1-x^2)(5+x)|\psi^D(0)|^2,
\end{aligned} \tag{5.14}$$

where the parameter η is introduced via

$$\langle\mathcal{B}_c|(\bar{c}c)(\bar{q}q)|\mathcal{B}_c\rangle = -\eta\langle\mathcal{B}_c|(\bar{c}q)(\bar{q}c)|\mathcal{B}_c\rangle, \tag{5.15}$$

so that $\eta = 1$ in the valence quark approximation. In the zero light quark mass limit ($x = 0$) and in the valence quark approximation, the reader can check that results of (5.14) are in agreement with those obtained in [38, 39, 52] except the Cabibbo-suppressed W-exchange contribution to Ω_c^0, $\Gamma^{\rm ann}(\Omega_c)$. We have a coefficient of 6 arising from the matrix element $\langle\Omega_c|(\bar{c}s)(\bar{s}c)|\Omega_c\rangle = -6|\psi^{\Omega_c}_{cs}(0)|^2(2m_{\Omega_c})$ [48], while the coefficient is claimed to be $\frac{10}{3}$ in [52].

Neglecting the small difference between r_{Λ_c}, r_{Ξ_c} and r_{Ω_c} and setting $x = 0$, the inclusive nonleptonic rates of charmed baryons in the valence quark approximation have the expressions:

$$\begin{aligned}
\Gamma_{\rm NL}(\Lambda_c^+) &= \Gamma^{\rm dec}(\Lambda_c^+) + \cos^2_C\Gamma^{\rm ann} + \Gamma^{\rm int}_- + \sin^2_C\Gamma^{\rm int}_+, \\
\Gamma_{\rm NL}(\Xi_c^+) &= \Gamma^{\rm dec}(\Xi_c^+) + \sin^2_C\Gamma^{\rm ann} + \Gamma^{\rm int}_- + \cos^2_C\Gamma^{\rm int}_+, \\
\Gamma_{\rm NL}(\Xi_c^0) &= \Gamma^{\rm dec}(\Xi_c^0) + \Gamma^{\rm ann} + \Gamma^{\rm int}_- + \Gamma^{\rm int}_+, \\
\Gamma_{\rm NL}(\Omega_c^0) &= \Gamma^{\rm dec}(\Omega_c^0) + 6\sin^2_C\Gamma^{\rm ann} + \frac{10}{3}\cos^2_C\Gamma^{\rm int}_+,
\end{aligned} \tag{5.16}$$

with θ_C being the Cabibbo angle.

Assuming the D meson wavefunction at the origin squared $|\psi^D_{c\bar{q}}(0)|^2$ being given by $\frac{1}{12}f_D^2 m_D$, we obtain $|\psi^{\Lambda_c}(0)|^2 = 7.5\times10^{-3}{\rm GeV}^3$ for $f_D = 220$ MeV.[4] To proceed to the numerical calculations, we use the Wilson coefficients $c_1(\mu) = 1.35$ and $c_2(\mu) = -0.64$ evaluated at the scale $\mu = 1.25$ GeV. Since $\eta = 1$ in the valence-quark approximation and since the wavefunction squared ratio r is evaluated using the quark model, it is reasonable to assume that the NQM and the valence-quark approximation are most reliable when the baryon matrix elements are evaluated at a typical hadronic scale $\mu_{\rm had}$. As shown in [54], the parameters η and r renormalized at two different scales are related via the renormalization group equation, from which we obtain $\eta(\mu) \simeq 0.74\eta(\mu_{\rm had}) \simeq 0.74$ and $r(\mu) \simeq 1.36\, r(\mu_{\rm had})$ [48].

The results of calculations are summarized in Table V. It is clear that the lifetime pattern

$$\tau(\Xi_c^+) > \tau(\Lambda_c^+) > \tau(\Xi_c^0) > \tau(\Omega_c^0) \tag{5.17}$$

is in accordance with experiment. This lifetime hierarchy is qualitatively understandable. The Ξ_c^+ baryon is longest-lived among charmed baryons because of the smallness of W-exchange and partial cancellation between constructive and destructive Pauli interferences, while Ω_c is shortest-lived due to the presence of two s quarks in the Ω_c that renders the contribution of $\Gamma^{\rm int}_+$ largely enhanced. From Eq. (5.14) we also see that $\Gamma^{\rm int}_+$ is always positive, $\Gamma^{\rm int}_-$ is negative and that the constructive interference is larger than the magnitude of the destructive one. This explains why $\tau(\Xi_c^+) > \tau(\Lambda_c^+)$. It is also clear from Table V that, although the qualitative feature of the lifetime pattern is comprehensive, the quantitative estimates of charmed baryon lifetimes and their ratios are still rather poor.

In [52], a much larger charmed baryon wave function at origin is employed. This is based on the argument originally advocated in [37]. The physical charmed meson decay constant f_D is related to the asymptotic static value F_D via

$$f_D = F_D\left(1 - \frac{|\mu|}{m_c} + \mathcal{O}(\frac{1}{m_c^2})\right). \tag{5.18}$$

[4] The recent CLEO measurement of $D^+\to\mu^+\nu$ yields $f_{D^+} = 222.6\pm16.7^{+2.8}_{-3.4}$ MeV [53].

TABLE V: Various contributions to the decay rates (in units of 10^{-12} GeV) of charmed baryons. The charmed meson wavefunction at the origin squared $|\psi^D(0)|^2$ is taken to be $\frac{1}{12} f_D^2 m_D$. Experimental values are taken from [3].

	$\Gamma^{\rm dec}$	$\Gamma^{\rm ann}$	$\Gamma_-^{\rm int}$	$\Gamma_+^{\rm int}$	$\Gamma_{\rm SL}$	$\Gamma^{\rm tot}$	$\tau(10^{-13}s)$	$\tau_{\rm expt}(10^{-13}s)$
Λ_c^+	1.006	1.342	-0.196		0.323	2.492	2.64	2.00 ± 0.06
Ξ_c^+	1.006	0.071	-0.203	0.364	0.547	1.785	3.68	4.42 ± 0.26
Ξ_c^0	1.006	1.466		0.385	0.547	3.404	1.93	$1.12^{+0.13}_{-0.10}$
Ω_c^0	1.132	0.439		1.241	1.039	3.851	1.71	0.69 ± 0.12

It was argued in [37] that one should not use the physical value of f_D when relating $|\psi^{\mathcal{B}_c}(0)|^2$ to $|\psi^D(0)|^2$ for reason of consistency since the widths have been calculated through order $1/m_c^3$ only. Hence, the part of f_D which is not suppressed by $1/m_c$ should not be taken into account. However, if we use $F_D \sim 2f_D$ for the wave function $|\psi^D(0)|^2$, we find that the predicted lifetimes of charmed baryons become too short compared to experiment except Ω_c^0. By contrast, using $|\psi^{\Lambda_c}(0)|^2 = 2.62 \times 10^{-2}\text{GeV}^3$ and the so-called hybrid renormalization, lifetimes $\tau(\Lambda_c^+) = 2.39$, $\tau(\Xi_c^+) = 2.51$, $\tau(\Xi_c^0) = 0.96$ and $\tau(\Omega_c^0) = 0.61$ in units of $10^{-13}s$ are obtained in [52]. They are in better agreement with the data except Ξ_c^+. The predicted ratio $\tau(\Xi_c^+)/\tau(\Lambda_c^+) = 1.05$ is too small compared to the experimental value of 2.21 ± 0.15. By inspecting Eq. (5.16), it seems to be very difficult to enhance the ratio by a factor of 2.

In short, when the lifetimes of charmed baryons are analyzed within the framework of the heavy quark expansion, the qualitative feature of the lifetime pattern is understandable, but a quantitative description of charmed baryon lifetimes is still lack. This may be ascribed to the following possibilities:

1. Unlike the semileptonic decays, the heavy quark expansion in inclusive nonleptonic decays cannot be justified by analytic continuation into the complex plane and local duality has to be assumed in order to apply the OPE directly in the physical region. The may suggest a significant violation of quark-hadron local duality in the charm sector.

2. Since the c quark is not heavy enough, it casts doubts on the validity of heavy quark expansion for inclusive charm decays. This point can be illustrated by the following example. It is well known that the observed lifetime difference between the D^+ and D^0 is ascribed to the destructive interference in D^+ decays and/or the constructive W-exchange contribution to D^0 decays. However, there is a serious problem with the evaluation of the destructive Pauli interference $\Gamma^{\rm int}(D^+)$ in D^+. A direct calculation analogous to $\Gamma_-^{\rm int}(\mathcal{B}_c)$ in the charmed baryon sector indicates that $\Gamma^{\rm int}(D^+)$ overcomes the c quark decay rate so that the resulting nonleptonic decay width of D^+ becomes negative [37, 55]. This certainly does not make sense. This example clearly indicates that the $1/m_c$ expansion in charm decay is not well convergent and sensible, to say the least. It is not clear if the situation is improved even after higher dimension terms are included.

3. To overcome the aforementioned difficulty with $\Gamma^{\rm int}(D^+)$, it has been conjectured in [37] that higher-dimension corrections amount to replacing m_c by m_D in the expansion parameter

$f_D^2 m_D/m_c^3$, so that it becomes f_D^2/m_D^2. As a consequence, the destructive Pauli interference will be reduced by a factor of $(m_c/m_D)^3$. By the same token, the Pauli interference in charmed baryon decay may also be subject to the same effect. Another way of alleviating the problem is to realize that the usual local four-quark operators are derived in the heavy quark limit so that the effect of spectator light quarks can be neglected. Since the charmed quark is not heavy enough, it is very important, as stressed by Chernyak [55], to take into account the nonzero momentum of spectator quarks in charm decay. In the framework of heavy quark expansion, this spectator effect can be regarded as higher order $1/m_c$ corrections.

4. One of the major theoretical uncertainties comes from the evaluation of the four-quark matrix elements. One can hope that lattice QCD will provide a better handle on those quantities.

VI. HADRONIC WEAK DECAYS

Contrary to the significant progress made over the last 20 years or so in the studies of the heavy meson weak decay, advancement in the arena of heavy baryons, both theoretical and experimental, has been relatively slow. This is partly due to the smaller baryon production cross section and the shorter lifetimes of heavy baryons. From the theoretical point of view, baryons being made out of three quarks, in contrast to two quarks for mesons, bring along several essential complications. First of all, the factorization approximation that the hadronic matrix element is factorized into the product of two matrix elements of single currents and that the nonfactorizable term such as the W-exchange contribution is negligible relative to the factorizable one is known empirically to be working reasonably well for describing the nonleptonic weak decays of heavy mesons. However, this approximation is *a priori* not directly applicable to the charmed baryon case as W-exchange there, manifested as pole diagrams, is no longer subject to helicity and color suppression. This is different from the naive color suppression of internal W-emission. It is known in the heavy meson case that nonfactorizable contributions will render the color suppression of internal W-emission ineffective. However, the W-exchange in baryon decays is not subject to color suppression even in the absence of nonfactorizable terms. A simple way to see this is to consider the large-N_c limit. Although the W-exchange diagram is down by a factor of $1/N_c$ relative to the external W-emission one, it is compensated by the fact that the baryon contains N_c quarks in the limit of large N_c, thus allowing N_c different possibilities for W exchange between heavy and light quarks [56]. That is, the pole contribution can be as important as the factorizable one. The experimental measurement of the decay modes $\Lambda_c^+ \to \Sigma^0\pi^+$, $\Sigma^+\pi^0$ and $\Lambda_c^+ \to \Xi^0 K^+$, which do not receive any factorizable contributions, indicates that W-exchange indeed plays an essential role in charmed baryon decays. Second, there are more possibilities in drawing the quark daigram amplitudes as depicted in Fig. 3; in general there exist two distinct internal W-emissions and several different W-exchange diagrams which will be discussed in more detail shortly.

Historically, the two-body nonleptonic weak decays of charmed baryons were first studied by utilizing the same technique of current algebra as in the case of hyperon decays [57]. However, the use of the soft-meson theorem makes sense only if the emitted meson is of the pseudoscalar type and its momentum is soft enough. Obviously, the pseudoscalar-meson final state in charmed bayon decay is far from being "soft". Therefore, it is not appropriate to make the soft meson limit. It is no longer justified to apply current algebra to heavy-baryon weak decays, especially for s-wave amplitudes. Thus one has to go back to the original pole model, which is nevertheless reduced to current algebra in the soft pseudoscalar-meson limit, to deal with nonfactorizable contributions. The merit of the pole model is obvious: Its use is very general and is not limited to the soft meson limit and to the pseudoscalar-meson final state. Of course, the price we have to pay is that it requires the knowledge of the negative-parity baryon poles for the parity-violating transition. This also explains why the theoretical study of nonleptonic decays of heavy baryons is much more difficult than the hyperon and heavy meson decays.

The nonfactorizable pole contributions to hadronic weak decays of charmed baryons have been studied in the literature [58–60]. In general, nonfactorizable s- and p-wave amplitudes for $\frac{1}{2}^+ \to \frac{1}{2}^+ + P(V)$ decays (P: pseudoscalar meson, V: vector meson), for example, are dominated by $\frac{1}{2}^-$ low-lying baryon resonances and $\frac{1}{2}^+$ ground-state baryon poles, respectively. However, the estimation of pole amplitudes is a difficult and nontrivial task since it involves weak baryon matrix elements and strong coupling constants of $\frac{1}{2}^+$ and $\frac{1}{2}^-$ baryon states. This is the case in particular for s-wave terms as we know very little about the $\frac{1}{2}^-$ states. As a consequence, the evaluation of pole diagrams is far more uncertain than the factorizable terms. In short, W-exchange plays a dramatic role in the charmed baryon case and it even dominates over the spectator contribution in hadronic decays of Λ_c^+ and Ξ_c^0 [61].

Since the light quarks of the charmed baryon can undergo weak transitions, one can also have charm-flavor-conserving weak decays, e.g., $\Xi_c \to \Lambda_c\pi$ and $\Omega_c \to \Xi_c\pi$, where the charm quark behaves as a spectator. This special class of weak decays usually can be calculated more reliably than the conventional charmed baryon weak decays.

A. Quark-diagram scheme

Besides dynamical model calculations, it is useful to study the nonleptonic weak decays in a way which is as model independent as possible. The two-body nonleptonic decays of charmed baryons have been analyzed in terms of SU(3)-irreducible-representation amplitudes [62, 63]. However, the quark-diagram scheme (i.e., analyzing the decays in terms of quark-diagram amplitudes) has the advantage that it is more intuitive and easier for implementing model calculations. It has been successfully applied to the hadronic weak decays of charmed and bottom mesons [64, 65]. It has provided a framework with which we not only can do the least-model-dependent data analysis and give predictions but also make evaluations of theoretical model calculations.

A general formulation of the quark-diagram scheme for the nonleptonic weak decays of charmed baryons has been given in [66] (see also [67]). The general quark diagrams shown in Fig. 3 are: the external W-emission tree diagram T, internal W-emission diagrams C and C', W-exchange diagrams E_1, E_2 and E' (see Fig. 2 of [66] for notation and for details). There are also penguin-type quark diagrams which are presumably negligible in charm decays due to GIM cancellation. The quark diagram amplitudes T, C, $C' \cdots$ etc. in each type of hadronic decays are in general not the same. For octet baryons in the final state, each of the W-exchange amplitudes has two more independent types: the symmetric and the antisymmetric, for example, E_{1A}, E_{2A}, E_{2S}, E'_A and E'_S [66]. The antiquark produced from the charmed quark decay $c \to q_1 q_2 \bar{q}_3$ in diagram C' can combine with q_1 or q_2 to form an outgoing meson. Consequently, diagram C' contains factorizable contributions but C does not. It should be stressed that all quark graphs used in this approach are topological with all the strong interactions included, i.e. gluon lines are included in all possible ways. Hence, they are *not* Feynman graphs. Moreover, final-state interactions are also classified

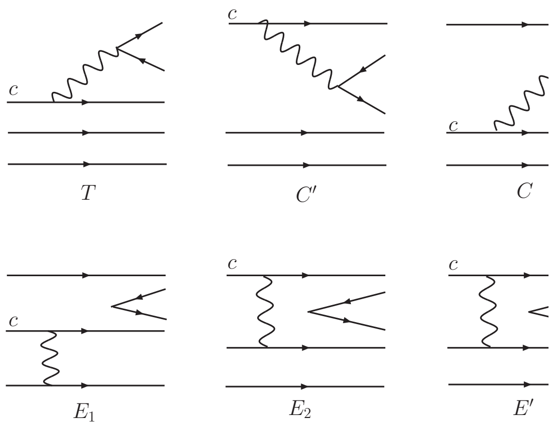

FIG. 3: Quark diagrams for charmed baryon decays

in the same manner. A good example is the reaction $D^0 \to \bar{K}^0\phi$, which can be produced via final-state rescattering even in the absence of the W-exchange diagram. Then it was shown in [65] that this rescattering diagram belongs to the generic W-exchange topology.

Since the two spectator light quarks in the heavy baryon are antisymmetrized in the antitriplet charmed baryon $\mathcal{B}_c(\bar{\mathbf{3}})$ and the wave function of the decuplet baryon $\mathcal{B}(\mathbf{10})$ is totally symmetric, it is clear that factorizable amplitudes T and C' cannot contribute to the decays of type $\mathcal{B}_c(\bar{\mathbf{3}}) \to \mathcal{B}(\mathbf{10}) + M(\mathbf{8})$; it receives contributions only from the W-exchange and penguin-type diagrams (see Fig. 1 of [66]). Examples are $\Lambda_c^+ \to \Delta^{++}K^-, \Sigma^{*+}\rho^0, \Sigma^{*+}\eta, \Xi^{*0}K^+$ and $\Xi_c^0 \to \Sigma^{*+}\bar{K}^0$. They can only proceed via W-exchange. Hence, the experimental observation of them implies that the W-exchange mechanism plays a significant role in charmed baryon decays. The quark diagram amplitudes for all two-body decays of (Cabibbo-allowed, singly suppressed and doubly suppressed) $\Lambda_c^+, \Xi_c^{+,0}$ and Ω_c^0 are listed in [66]. In the SU(3) limit, there exist many relations among various charmed baryon decay amplitudes, see [66] for detail. For charmed baryon decays, there are only a few decay modes which proceed through factorizable external or internal W-emission diagram, namely, Cabibbo-allowed $\Omega_c^0 \to \Omega^-\pi^+(\rho^+)$, $\Xi^{*0}\bar{K}^0(\bar{K}^{*0})$ and Cabibbo-suppressed $\Lambda_c^+ \to p\phi$.

B. Dynamical model calculation

To proceed we first consider the Cabibbo-allowed decays $\mathcal{B}_c(\frac{1}{2}^+) \to \mathcal{B}(\frac{1}{2}^+) + P(V)$. The general amplitudes are

$$
\begin{aligned}
M[\mathcal{B}_i(1/2^+) \to \mathcal{B}_f(1/2^+) + P] &= i\bar{u}_f(p_f)(A + B\gamma_5)u_i(p_i), \qquad (6.1)\\
M[\mathcal{B}_i(1/2^+) \to \mathcal{B}_f(1/2^+) + V] &= \bar{u}_f(p_f)\varepsilon^{*\mu}[A_1\gamma_\mu\gamma_5 + A_2(p_f)_\mu\gamma_5 + B_1\gamma_\mu + B_2(p_f)_\mu]u_i(p_i),
\end{aligned}
$$

where ε_μ is the polarization vector of the vector meson, A, (B, B_1, B_2) and A_2 are s-wave, p-wave and d-wave amplitudes, respectively, and A_1 consists of both s-wave and d-wave ones. The QCD-corrected weak Hamiltonian responsible for Cabibbo-allowed hadronic decays of charmed baryons reads

$$
\mathcal{H}_W = \frac{G_F}{\sqrt{2}} V_{cs}V_{ud}^*(c_1O_1 + c_2O_2), \qquad (6.2)
$$

where $O_1 = (\bar{s}c)(\bar{u}d)$ and $O_2 = (\bar{s}d)(\bar{u}c)$ with $(\bar{q}_1 q_2) \equiv \bar{q}_1\gamma_\mu(1-\gamma_5)q_2$. ¿From the expression of $O_{1,2}$, it is clear that factorization occurs if the final-state meson is $\pi^+(\rho^+)$ or $\bar{K}^0(\bar{K}^{*0})$. Explicitly,

$$
\begin{aligned}
A^{\rm fac}(\mathcal{B}_i \to \mathcal{B}_f + \pi^+) &= \lambda\, a_1 f_P (m_i - m_f) f_1(m_\pi^2), \\
B^{\rm fac}(\mathcal{B}_i \to \mathcal{B}_f + \pi^+) &= \lambda\, a_1 f_P (m_i + m_f) g_1(m_\pi^2), \\
A^{\rm fac}(\mathcal{B}_i \to \mathcal{B}_f + \bar{K}^0) &= \lambda\, a_2 f_P (m_i - m_f) f_1(m_K^2), \\
B^{\rm fac}(\mathcal{B}_i \to \mathcal{B}_f + \bar{K}^0) &= \lambda\, a_2 f_P (m_i + m_f) g_1(m_K^2),
\end{aligned} \tag{6.3}
$$

and

$$
\begin{aligned}
A_1^{\rm fac}(\mathcal{B}_i \to \mathcal{B}_f + \rho^+) &= -\lambda\, a_1 f_\rho m_\rho [g_1(m_\rho^2) + g_2(m_\rho^2)(m_i - m_f)], \\
A_2^{\rm fac}(\mathcal{B}_i \to \mathcal{B}_f + \rho^+) &= -2\lambda\, a_1 f_\rho m_\rho g_2(m_\rho^2), \\
B_1^{\rm fac}(\mathcal{B}_i \to \mathcal{B}_f + \rho^+) &= \lambda\, a_1 f_\rho m_\rho [f_1(m_\rho^2) - f_2(m_\rho^2)(m_i + m_f)], \\
B_2^{\rm fac}(\mathcal{B}_i \to \mathcal{B}_f + \rho^+) &= 2\lambda\, a_1 f_\rho m_\rho f_2(m_\rho^2),
\end{aligned}
$$

and similar expressions for $\mathcal{B}_i \to \mathcal{B}_f + \bar{K}^{*0}$, where $\lambda = G_F V_{cs} V_{ud}^*/\sqrt{2}$, f_i and g_i are the form factors defined by $(q = p_i - p_f)$

$$
\begin{aligned}
\langle \mathcal{B}_f(p_f)|V_\mu - A_\mu|\mathcal{B}_i(p_i)\rangle &= \bar{u}_f(p_f)[f_1(q^2)\gamma_\mu + i f_2(q^2)\sigma_{\mu\nu}q^\nu + f_3(q^2)q_\mu \\
&\quad -(g_1(q^2)\gamma_\mu + i g_2(q^2)\sigma_{\mu\nu}q^\nu + g_3(q^2)q_\mu)\gamma_5]u_i(p_i).
\end{aligned} \tag{6.4}
$$

In the naive factorization approach, the coefficients a_1 for the external W-emission amplitude and a_2 for internal W-emission are given by $(c_1 + \frac{c_2}{N_c})$ and $(c_2 + \frac{c_1}{N_c})$, respectively. However, we have learned from charmed meson decays that the naive factorization approach never works for the decay rate of color-suppressed decay modes, though it usually operates for color-allowed decays. Empirically, it was learned in the 1980s that if the Fierz-transformed terms characterized by $1/N_c$ are dropped, the discrepancy between theory and experiment is greatly improved [68]. This leads to the so-called large-N_c approach for describing hadronic D decays [69]. Theoretically, explicit calculations based on the QCD sum-rule analysis [70] indicate that the Fierz terms are indeed largely compensated by the nonfactorizable corrections.

As the discrepancy between theory and experiment for charmed meson decays gets much improved in the $1/N_c$ expansion method, it is natural to ask if this scenario also works in the baryon sector? This issue can be settled down by the experimental measurement of the Cabibbo-suppressed mode $\Lambda_c^+ \to p\phi$, which receives contributions only from the factorizable diagrams. As pointed out in [58], the large-N_c predicted rate is in good agreement with the measured value. By contrast, its decay rate prdicted by the naive factorization approximation is too small by a factor of 15. Therefore, the $1/N_c$ approach also works for the factorizable amplitude of charmed baryon decays. This also implies that the inclusion of nonfactorizable contributions is inevitable and necessary. If nonfactorizable effects amount to a redefinition of the effective parameters a_1, a_2 and are universal (i.e., channel-independent) in charm decays, then we still have a new factorization scheme with the universal parameters a_1, a_2 to be determined from experiment. Throughout this paper, we will thus treat a_1 and a_2 as free effective parameters.

At the hadronic level, the decay amplitudes for quark diagrams T and $C'$are conventionally evaluated using the factorization approximation. How do we tackle with the remaining nonfactorizable diagrams $C,\ E_1,\ E_2,\ E'$? One popular approach is to consider the contributions from all possible intermediate states. Among all possible pole contributions, including resonances and continuum states, one usually focuses on the most important poles such as the low-lying $\frac{1}{2}^+, \frac{1}{2}^-$ states, known

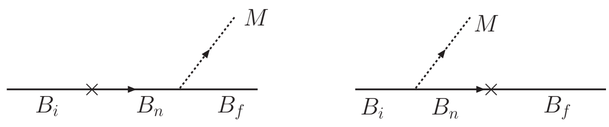

FIG. 4: Pole diagrams for charmed baryon decay $\mathcal{B}_i \to \mathcal{B}_f + M$.

as pole approximation. More specifically, the s-wave amplitude is dominated by the low-lying $1/2^-$ resonances and the p-wave one governed by the ground-state $1/2^+$ poles (see Fig. 4):

$$\begin{aligned} A^{\rm nf} &= -\sum_{\mathcal{B}_n^*(1/2^-)} \left(\frac{g_{\mathcal{B}_f\mathcal{B}_{n^*}P}\, b_{n^*i}}{m_i - m_{n^*}} + \frac{b_{fn^*}\, g_{\mathcal{B}_{n^*}\mathcal{B}_iP}}{m_f - m_{n^*}} \right) + \cdots, \\ B^{\rm nf} &= -\sum_{\mathcal{B}_n} \left(\frac{g_{\mathcal{B}_f\mathcal{B}_nP}\, a_{ni}}{m_i - m_n} + \frac{a_{fn}\, g_{\mathcal{B}_n\mathcal{B}_iP}}{m_f - m_n} \right) + \cdots, \end{aligned} \tag{6.5}$$

where $A^{\rm nf}$ and $B^{\rm nf}$ are the nonfactorizable s- and p-wave amplitudes of $\mathcal{B}_c \to \mathcal{B}P$, respectively, ellipses in Eq.(6.5) denote other pole contributions which are negligible for our purposes, and a_{ij} as well as b_{i^*j} are the baryon-baryon matrix elements defined by

$$\langle \mathcal{B}_i|\mathcal{H}_W|\mathcal{B}_j\rangle = \bar{u}_i(a_{ij} - b_{ij}\gamma_5)u_j, \qquad \langle \mathcal{B}_i^*(1/2^-)|\mathcal{H}_W^{\rm pv}|\mathcal{B}_j\rangle = i b_{i^*j}\, \bar{u}_i u_j, \tag{6.6}$$

with $b_{ji^*} = -b_{i^*j}$. Evidently, the calculation of s-wave amplitudes is more difficult than the p-wave owing to the troublesome negative-parity baryon resonances which are not well understood in the quark model. In [58, 59], the form factors appearing in factorizable amplitudes and the strong coupling constants and baryon transition matrix elements relevant to nonfactorizable contributions are evaluated using the MIT bag model [71]. Two of the pole model calculations for branching ratios [59, 60] are displayed in Table VI. The study of charmed baryon hadronic decays in [72] is similar to [59, 60] except that the effect of W-exchange is parametrized in terms of the baryon wave function at origin. Sharma and Verma [72] defined a parameter $r = |\psi^{\mathcal{B}_c}(0)|^2/|\psi^{\mathcal{B}}(0)|^2$ and argued that its value is close to 1.4 . A variant of the pole model has been considered in [73] in which the effects of pole-model-induced SU(4) symmetry breaking in parity-conserving and parity-violating amplitudes are studied.

Instead of decomposing the decay amplitude into products of strong couplings and two-body weak transitions, Körner and Krämer [56] have analyzed the nonleptonic weak processes using the spin-flavor structure of the effective Hamiltonian and the wave functions of baryons and mesons described by the covariant quark model. The nonfactorizable amplitudes are then obtained in terms of two wave function overlap parameters H_2 and H_3, which are in turn determined by fitting to the experimental data of $\Lambda_c^+ \to p\bar{K}^0$ and $\Lambda_c^+ \to \Lambda\pi^+$, respectively. Despite the absence of first-principles calculation of the parameters H_2 and H_3, this quark model approach has fruitful predictions for not only $\mathcal{B}_c \to \mathcal{B} + P$, but also $\mathcal{B}_c \to \mathcal{B} + V$, $\mathcal{B}^*(3/2^+) + P$ and $\mathcal{B}^*(3/2^+) + V$ decays. Another advantage of this analysis is that each amplitude has one-to-one quark-diagram interpretation. While the overlap integrals are treated as phenomenological parameters to be determined from a fit to the data, Ivanov *et al.* [74] developed a microscopic approach to the overlap integrals by specifying the form of the hadron-quark transition vertex including the explicit momentum dependence of the Lorentz scalar part of this vertex.

TABLE VI: Branching ratios of Cabibbo-allowed $\mathcal{B}_c \to \mathcal{B} + P$ decays in various models. Results of [56, 59, 60] have been normalized using the current world averages of charmed baryon lifetimes [3]. Branching ratios cited from [72] are for $\phi_{\eta-\eta'} = -23°$ and $r = 1.4$.

Decay	Körner, Krämer [56]	Xu, Kamal [60]	Cheng, Tseng [59]	Ivanov et al. [74]	Żenczykowski [73]	Sharma, Verma [72]	Expt. [3]
$\Lambda_c^+ \to \Lambda\pi^+$	input	1.62	0.88	0.79	0.54	1.12	0.90± 0.28
$\Lambda_c^+ \to \Sigma^0\pi^+$	0.32	0.34	0.72	0.88	0.41	1.34	0.99± 0.32
$\Lambda_c^+ \to \Sigma^+\pi^0$	0.32	0.34	0.72	0.88	0.41	1.34	1.00± 0.34
$\Lambda_c^+ \to \Sigma^+\eta$	0.16			0.11	0.94	0.57	0.48± 0.17
$\Lambda_c^+ \to \Sigma^+\eta'$	1.28			0.12	0.12	0.10	
$\Lambda_c^+ \to p\bar{K}^0$	input	1.20	1.26	2.06	1.79	1.64	2.3± 0.6
$\Lambda_c^+ \to \Xi^0 K^+$	0.26	0.10		0.31	0.36	0.13	0.39± 0.14
$\Xi_c^+ \to \Sigma^+\bar{K}^0$	6.45	0.44	0.84	3.08	1.56	0.04	
$\Xi_c^+ \to \Xi^0\pi^+$	3.54	3.36	3.93	4.40	1.59	0.53	0.55 ± 0.16^a
$\Xi_c^0 \to \Lambda\bar{K}^0$	0.12	0.37	0.27	0.42	0.35	0.54	seen
$\Xi_c^0 \to \Sigma^0\bar{K}^0$	1.18	0.11	0.13	0.20	0.11	0.07	
$\Xi_c^0 \to \Sigma^+ K^-$	0.12	0.12		0.27	0.36	0.12	
$\Xi_c^0 \to \Xi^0\pi^0$	0.03	0.56	0.28	0.04	0.69	0.87	
$\Xi_c^0 \to \Xi^0\eta$	0.24			0.28	0.01	0.22	
$\Xi_c^0 \to \Xi^0\eta'$	0.85			0.31	0.09	0.06	
$\Xi_c^0 \to \Xi^-\pi^+$	1.04	1.74	1.25	1.22	0.61	2.46	seen
$\Omega_c^0 \to \Xi^0\bar{K}^0$	1.21		0.09	0.02			

[a]Branching ratio relative to $\Xi_c^+ \to \Xi^-\pi^+\pi^+$.

C. Discussions

Various model predictions of the branching ratios and decay asymmetries for Cabibbo-allowed $\mathcal{B}_c \to \mathcal{B} + P(V)$ decays are summarized in Tables VI-IX. In the following we shall first discuss the decay asymmetry parameter α and then turn to the decay rates.

1. Decay asymmetry

A very useful information is provided by the study of the polarization of the daughter baryon $\mathcal{B}'$ in the decay $\mathcal{B} \to \mathcal{B}'\pi$. Its general expression is given by

$$\mathbf{P}_{\mathcal{B}'} = \frac{(\alpha_{\mathcal{B}} + \mathbf{P}_{\mathcal{B}} \cdot \mathbf{n})\mathbf{n} + \beta_{\mathcal{B}}(\mathbf{n} \times \mathbf{P}_{\mathcal{B}}) + \gamma_{\mathcal{B}}\mathbf{n} \times (\mathbf{n} \times \mathbf{P}_{\mathcal{B}})}{1 + \alpha_{\mathcal{B}}\mathbf{P}_{\mathcal{B}} \cdot \mathbf{n}}, \tag{6.7}$$

where $\mathbf{P}_{\mathcal{B}}$ is the parent baryon polarization, $\alpha_{\mathcal{B}}$, $\beta_{\mathcal{B}}$ and $\gamma_{\mathcal{B}}$ are the parent baryon asymmetry parameters and $\mathbf{n}$ is a unit vector along the daughter baryon $\mathcal{B}'$ in the parent baryon frame. If

TABLE VII: The predicted asymmetry parameter α for Cabibbo-allowed $\mathcal{B}_c \to \mathcal{B} + P$ decays in various models. Results cited from [72] are for $\phi_{\eta-\eta'} = -23^\circ$ and $r = 1.4$.

Decay	Körner, Krämer [56]	Xu, Kamal [60]	Cheng, Tseng [59]	Ivanov et al. [74]	Żenczykowski [73]	Sharma, Verma [72]	Expt. [3]
$\Lambda_c^+ \to \Lambda\pi^+$	−0.70	−0.67	−0.95	−0.95	−0.99	−0.99	-0.91 ± 0.15
$\Lambda_c^+ \to \Sigma^0\pi^+$	0.70	0.92	0.78	0.43	0.39	−0.31	
$\Lambda_c^+ \to \Sigma^+\pi^0$	0.71	0.92	0.78	0.43	0.39	−0.31	-0.45 ± 0.32
$\Lambda_c^+ \to \Sigma^+\eta$	0.33			0.55	0	−0.91	
$\Lambda_c^+ \to \Sigma^+\eta'$	−0.45			−0.05	−0.91	0.78	
$\Lambda_c^+ \to p\bar{K}^0$	−1.0	0.51	−0.49	−0.97	−0.66	−0.99	
$\Lambda_c^+ \to \Xi^0 K^+$	0	0		0	0	0	
$\Xi_c^+ \to \Sigma^+\bar{K}^0$	−1.0	0.24	−0.09	−0.99	1.00	0.54	
$\Xi_c^+ \to \Xi^0\pi^+$	−0.78	−0.81	−0.77	−1.0	1.00	−0.27	
$\Xi_c^0 \to \Lambda\bar{K}^0$	−0.76	1.0	−0.73	−0.75	−0.29	−0.79	
$\Xi_c^0 \to \Sigma^0\bar{K}^0$	−0.96	−0.99	−0.59	−0.55	−0.50	0.48	
$\Xi_c^0 \to \Sigma^+K^-$	0	0		0	0	0	
$\Xi_c^0 \to \Xi^0\pi^0$	0.92	0.92	−0.54	0.94	0.21	−0.80	
$\Xi_c^0 \to \Xi^0\eta$	−0.92			−1.0	−0.04	0.21	
$\Xi_c^0 \to \Xi^0\eta'$	−0.38			−0.32	−1.00	0.80	
$\Xi_c^0 \to \Xi^-\pi^+$	−0.38	−0.38	−0.99	−0.84	−0.79	−0.97	-0.6 ± 0.4
$\Omega_c^0 \to \Xi^0\bar{K}^0$	0.51		−0.93	−0.81			

the parent baryon is unpolarized, the above equation reduces to $\mathbf{P}_{\mathcal{B}'} = \alpha_{\mathcal{B}}\mathbf{n}$, which implies that the baryon $\mathcal{B}'$ obtained from the decay of the unpolarized baryon $\mathcal{B}$ is longitudinally polarized by the amount of $\alpha_{\mathcal{B}}$. The transverse polarization components are measured by the parameters $\beta_{\mathcal{B}}$ and $\gamma_{\mathcal{B}}$. In terms of the s- and p-wave amplitudes in Eq. (6.1), the baryon parameters have the expressions

$$\alpha = \frac{2\mathrm{Re}(S^*P)}{|S|^2+|P|^2}, \qquad \beta = \frac{2\mathrm{Im}(S^*P)}{|S|^2+|P|^2}, \qquad \gamma = \frac{|S|^2-|P|^2}{|S|^2+|P|^2}, \tag{6.8}$$

where

$$S = \sqrt{2m_{\mathcal{B}'}(E'+m_{\mathcal{B}'})}\, A, \qquad P = \sqrt{2m_{\mathcal{B}'}(E'-m_{\mathcal{B}'})}\, B. \tag{6.9}$$

When CP is conserved and final-state interactions are negligible, β vanishes. Since the sign of $\alpha_{\mathcal{B}}$ depends on the relative sign between s- and p-wave amplitudes, the measurement of α can be used to discriminate between different models.

The model predictions for the decay asymmetry α in $\Lambda_c^+ \to \Lambda\pi^+$ range from -0.67 to -0.99 (see Table VII). The current world average of α is -0.91 ± 0.15 [3], while the most recent measurement is $-0.78 \pm 0.16 \pm 0.19$ by FOCUS [75]. The agreement between theory and experiment implies the $V-A$ structure of the decay process $\Lambda_c^+ \to \Lambda\pi^+$.

TABLE VIII: Branching ratios of Cabibbo-allowed $\mathcal{B}_c \to \mathcal{B} + V$ decays in various models. The experimental value denoted by the superscript $*$ is the branching ratio relative to $\Xi_c^+ \to \Xi^-\pi^+\pi^+$.

Decay	Körner, Krämer [56]	Żenczykowski [73]	Cheng, Tseng [58]	Experiment [3]
$\Lambda_c^+ \to \Lambda\rho^+$	19.08	1.80	2.6	< 5
$\Lambda_c^+ \to \Sigma^0\rho^+$	3.14	1.56	0.19	0.99$\pm$ 0.32
$\Lambda_c^+ \to \Sigma^+\rho^0$	3.12	1.56	0.19	< 1.4
$\Lambda_c^+ \to \Sigma^+\omega$	4.02	1.10		2.7$\pm$ 1.0
$\Lambda_c^+ \to \Sigma^+\phi$	0.26	0.11		0.32$\pm$ 0.10
$\Lambda_c^+ \to p\bar{K}^{*0}$	3.08	5.03	3.3	1.6$\pm$ 0.5
$\Lambda_c^+ \to \Xi^0 K^{*+}$	0.12	0.11		0.39$\pm$ 0.14
$\Xi_c^+ \to \Sigma^+\bar{K}^{*0}$	2.34	7.38		$0.81 \pm 0.15^*$
$\Xi_c^+ \to \Xi^0\rho^+$	95.83	5.48		
$\Xi_c^0 \to \Lambda\bar{K}^{*0}$	1.12	1.15		
$\Xi_c^0 \to \Sigma^0\bar{K}^{*0}$	0.62	0.77		
$\Xi_c^0 \to \Sigma^+ K^{*-}$	0.39	0.37		
$\Xi_c^0 \to \Xi^0\rho^0$	1.71	1.22		
$\Xi_c^0 \to \Xi^0\omega$	2.33	0.15		
$\Xi_c^0 \to \Xi^0\phi$	0.18	0.10		
$\Xi_c^0 \to \Xi^-\rho^+$	12.29	1.50		
$\Omega_c^0 \to \Xi^0\bar{K}^{*0}$	0.59			

It is evident from Table VII that all the models except one model in [72] predict a positive decay asymmetry for the decay $\Lambda_c^+ \to \Sigma^+\pi^0$. Therefore, the measurement of $\alpha = -0.45 \pm 0.31 \pm 0.06$ by CLEO [76] is a big surprise. If the negative sign of α is confirmed in the future, this will imply an opposite sign between s-wave and p-wave amplitudes for this decay, contrary to the model expectation. The implication of this has been discussed in detail in [58]. Since the error of the previous CLEO measurement is very large, it is crucial to have more accurate measurements of the decay asymmetry for $\Lambda_c^+ \to \Sigma^+\pi^0$.

The decays $\Lambda_c^+ \to \Xi^0 K^+$ and $\Xi_c^0 \to \Sigma^+ K^-$ share some common features that they can proceed via W-exchange [66] and that their s-wave amplitudes are very small. As a consequence, their decay asymmetries are expected to be very tiny. Indeed, all the existing models predict vanishing s-wave amplitude and hence $\alpha = 0$ (cf. Table VII).

2. Λ_c^+ decays

Experimentally, nearly all the branching ratios of the Λ_c^+ are measured relative to the $pK^-\pi^+$ mode. Some Cabibbo-suppressed modes such as $\Lambda_c^+ \to \Lambda K^+$ and $\Lambda_c^+ \to \Sigma^0 K^+$ have been recently measured by BaBar [79]. Theoretically, only one model [80] gives predictions for the Cabibbo-

TABLE IX: Branching ratios and decay asymmetries (in parentheses) of Cabibbo-allowed $\mathcal{B}_c \to \mathcal{B}(3/2^+) + P(V)$ decays in various models. Experimental values denoted by the superscript $*$ are the branching ratios relative to $\Xi_c^+ \to \Xi^-\pi^+\pi^+$. The model calculations of Xu and Kamal are done in two different schemes [77].

Decay	Körner, Krämer [56]	Xu & Kamal [77]	Cheng [78]	Experiment [3]
$\Lambda_c^+ \to \Delta^{++}K^-$	2.70	1.00(0.00); 1.04(0.43)		0.86 ± 0.30
$\Lambda_c^+ \to \Delta^+\bar{K}^0$	0.90	0.34(0.00); 0.34(0.43)		0.99 ± 0.32
$\Lambda_c^+ \to \Xi^{*0}K^+$	0.50	0.08(0.00); 0.08(0.25)		0.26 ± 0.10
$\Lambda_c^+ \to \Sigma^{*+}\pi^0$	0.50	0.22(0.00); 0.24(0.40)		
$\Lambda_c^+ \to \Sigma^{*0}\pi^+$	0.50	0.22(0.00); 0.24(0.40)		
$\Lambda_c^+ \to \Sigma^{*+}\eta$	0.54			0.85 ± 0.33
$\Xi_c^+ \to \Sigma^{*+}\bar{K}^0$	0	0		$1.0 \pm 0.5^*$
$\Xi_c^+ \to \Xi^{*0}\pi^+$	0	0		$< 0.1^*$
$\Xi_c^0 \to \Omega^-K^+$	0.34	0.15(0.00); 0.16(0.27)		seen
$\Xi_c^0 \to \Sigma^{*0}\bar{K}^0$	0.25	0.09(0.00); 0.10(0.43)		
$\Xi_c^0 \to \Sigma^{*+}K^-$	0.49	0.18(0.00); 0.19(0.43)		
$\Xi_c^0 \to \Xi^{*0}\pi^0$	0.28	0.12(0.00); 0.13(0.40)		
$\Xi_c^0 \to \Xi^{*-}\pi^+$	0.56	0.25(0.00); 0.27(0.40)		
$\Omega_c^0 \to \Omega^-\pi^+$	$0.35a_1^2$	$1.47a_1^2(0)$; $1.44a_1^2(0)$	$0.92a_1^2(0.17)$	seen
$\Omega_c^0 \to \Xi^{*0}\bar{K}^0$	$0.40a_2^2$	$0.69a_2^2(0)$; $0.61a_2^2(0)$	$1.06a_2^2(0.35)$	
$\Omega_c^0 \to \Omega^-\rho^+$	$2.02a_1^2$	$8.02a_1^2(-0.08)$; $7.82a_2^2(-0.21)$	$3.23a_1^2(0.43)$	
$\Omega_c^0 \to \Xi^{*0}\bar{K}^{*0}$	$2.28a_2^2$	$3.15a_2^2(-0.09)$; $1.13a_2^2(-0.27)$	$1.60a_2^2(0.28)$	

suppressed decays.

The first measured Cabibbo-suppressed mode $\Lambda_c^+ \to p\phi$ is of particular interest because it receives contributions only from the factorizable diagram and is expected to be color suppressed in the naive factorization approach. An updated calculation in [81] yields

$$\mathcal{B}(\Lambda_c^+ \to p\phi) = 2.26 \times 10^{-3}a_2^2, \qquad \alpha(\Lambda_c^+ \to p\phi) = -0.10\,. \tag{6.10}$$

¿From the experimental measurement $\mathcal{B}(\Lambda_c^+ \to p\phi) = (8.2 \pm 2.7) \times 10^{-4}$ [3], it follows that

$$|a_2|_{\rm expt} = 0.60 \pm 0.10\,. \tag{6.11}$$

This is in excellent agreement with the $1/N_c$ approach where $a_2 = c_2(m_c) = -0.59$.

3. Ξ_c^+ decays

No absolute branching ratios have been measured. The branching ratios listed in Tables VI and VIII are the ones relative to $\Xi_c^+ \to \Xi^-\pi^+\pi^+$. Several Cabibbo-suppressed decay modes such as $p\bar{K}^{*0}$, $\Sigma^+\phi$ and $\Xi(1690)K^+$ have been observed.

The Cabibbo-allowed decays $\Xi_c^+ \to \mathcal{B}(3/2^+) + P$ have been studied and they are believed to be forbidden as they do not receive factorizable and $1/2^\pm$ pole contributions [56, 77]. However, the $\Sigma^{*+}\bar{K}^0$ mode was seen by FOCUS before [82] and this may indicate the importance of pole contributions beyond low-lying $1/2^\pm$ intermediate states.

4. Ξ_c^0 decays

No absolute branching ratios have been measured so far. However, there are several measurements of ratios of branching fractions, for example [3],

$$R_1 = \frac{\Gamma(\Xi_c^0 \to \Lambda K_S^0)}{\Gamma(\Xi_c^0 \to \Xi^-\pi^+)} = 0.21 \pm 0.02 \pm 0.02, \quad R_2 = \frac{\Gamma(\Xi_c^0 \to \Omega^- K^+)}{\Gamma(\Xi_c^0 \to \Xi^-\pi^+)} = 0.297 \pm 0.024\,. \tag{6.12}$$

Most models predict a ratio of R_1 smaller than 0.18 except the model of Żenczykowski [73] which yields $R_1 = 0.29$ due to the small predicted rate of $\Xi_c^0 \to \Xi^-\pi^+$ (see Table VI). The model of Körner and Krämer [56] predicts $R_2 = 0.33$ (Table IX), in agreement with experiment, but its prediction $R_1 = 0.06$ is too small compared to the data.

5. Ω_c^0 decays

One of the unique features of the Ω_c^0 decay is that the decay $\Omega_c^0 \to \Omega^-\pi^+$ proceeds only via external W-emission, while $\Omega_c^0 \to \Xi^{*0}\bar{K}^0$ proceeds via the factorizable internal W-emission diagram C'. The general amplitudes for $\mathcal{B}_c \to B^*(\frac{3}{2}^+) + P(V)$ are:

$$\begin{aligned} M[\mathcal{B}_i \to \mathcal{B}_f^*(3/2^+) + P] &= iq_\mu \bar{u}_f^\mu(p_f)(C + D\gamma_5)u_i(p_i), \\ M[\mathcal{B}_i \to \mathcal{B}_f^*(3/2^+) + V] &= \bar{u}_f^\nu(p_f)\varepsilon^{*\mu}[g_{\nu\mu}(C_1 + D_1\gamma_5) \\ &\quad + p_{1\nu}\gamma_\mu(C_2 + D_2\gamma_5) + p_{1\nu}p_{2\mu}(C_3 + D_3\gamma_5)]u_i(p_i), \end{aligned} \tag{6.13}$$

with u^μ being the Rarita-Schwinger vector spinor for a spin-$\frac{3}{2}$ particle. Various model predictions of Cabibbo-allowed $\Omega_c^0 \to \mathcal{B}(3/2^+) + P(V)$ are displayed in Table IX with the unknown parameters a_1 and a_2. From the decay $\Lambda_c^+ \to p\phi$ we learn that $|a_2| = 0.60 \pm 0.10$. Notice a sign difference of α for $\Omega_c \to \frac{3}{2}^+ + V$ in [77] and [78]. It seems to us that the sign of A_i and B_i in Eq. (58) of [77] should be flipped. A consequence of this sign change will render α positive in $\Omega_c \to \frac{3}{2}^+ + V$ decay. In the model of Xu and Kamal [77], the D-wave amplitude in Eq. (6.13) and hence the parameter α vanishes in the decay $\Omega_c \to \frac{3}{2}^+ + P$ due to the fact that the vector current is conserved at all q^2 in their scheme 1 and at $q^2 = 0$ in scheme 2.

D. Charm-flavor-conserving weak decays

There is a special class of weak decays of charmed baryons which can be studied in a reliable way, namely, heavy-flavor-conserving nonleptonic decays. Some examples are the singly Cabibbo-suppressed decays $\Xi_c \to \Lambda_c\pi$ and $\Omega_c \to \Xi_c'\pi$. The idea is simple: In these decays only the light quarks inside the heavy baryon will participate in weak interactions; that is, while the two light quarks undergo weak transitions, the heavy quark behaves as a "spectator". As the emitted light mesons are soft, the $\Delta S = 1$ weak interactions among light quarks can be handled by the well

TABLE X: Predicted semileptonic decay rates (in units of $10^{10}s^{-1}$) and decay asymmetries (second entry) in various models. Dipole q^2 dependence for form factors is assumed whenever the form-factor momentum dependence is not calculated in the model. Predictions of [85] are obtained in the non-relativistic quark model and the MIT bag model (in parentheses).

Process	Pérez-Marcial et al. [85]	Singleton [86]	Cheng, Tseng [81]	Ivanov et al. [87]	Luo [88]	Marques de Carvalho et al. [89]	Huang, Wang [90]	Expt. [3]
$\Lambda_c^+ \to \Lambda^0 e^+ \nu_e$	11.2 (7.7)	9.8	7.1	7.22	7.0	13.2 ± 1.8	10.9 ± 3.0	10.5 ± 3.0
				-0.812		-1	-0.88 ± 0.03	-0.86 ± 0.04
$\Xi_c^0 \to \Xi^- e^+ \nu_e$	18.1 (12.5)	8.5	7.4	8.16	9.7			seen
$\Xi_c^+ \to \Xi^0 e^+ \nu_e$	18.4 (12.7)	8.5	7.4	8.16	9.7			seen

known short-distance effective Hamiltonian. The synthesis of heavy-quark and chiral symmetries provides a natural setting for investigating these reactions [83]. The weak decays $\Xi_Q \to \Lambda_Q \pi$ with $Q = c, b$ were also studied in [84].

The combined symmetries of heavy and light quarks severely restricts the weak interactions allowed. In the symmetry limit, it is found that there cannot be $\mathcal{B}_{\bar{3}} - \mathcal{B}_6$ and $\mathcal{B}_6^* - \mathcal{B}_6$ nonleptonic weak transitions. Symmetries alone permit three types of transitions: $\mathcal{B}_{\bar{3}} - \mathcal{B}_{\bar{3}}$, $\mathcal{B}_6 - \mathcal{B}_6$ and $\mathcal{B}_6^* - \mathcal{B}_6$ transitions. However, in both the MIT bag and diquark models, only $\mathcal{B}_{\bar{3}} - \mathcal{B}_{\bar{3}}$ transitions have nonzero amplitudes.

The predicted rates are [83]

$$\Gamma(\Xi_c^0 \to \Lambda_c^+ \pi^-) = 1.7 \times 10^{-15}\,\mathrm{GeV}, \qquad \Gamma(\Xi_c^+ \to \Lambda_c^+ \pi^0) = 1.0 \times 10^{-15}\,\mathrm{GeV},$$
$$\Gamma(\Omega_c^0 \to \Xi_c'^+ \pi^-) = 4.3 \times 10^{-17}\,\mathrm{GeV}, \tag{6.14}$$

and the corresponding branching ratios are

$$\mathcal{B}(\Xi_c^0 \to \Lambda_c^+ \pi^-) = 2.9 \times 10^{-4}, \qquad \mathcal{B}(\Xi_c^+ \to \Lambda_c^+ \pi^0) = 6.7 \times 10^{-4},$$
$$\mathcal{B}(\Omega_c^0 \to \Xi_c'^+ \pi^-) = 4.5 \times 10^{-6}. \tag{6.15}$$

As stated above, the $\mathcal{B}_6 - \mathcal{B}_6$ transition $\Omega_c^0 \to \Xi_c'^+ \pi^-$ vanishes in the chiral limit. It receives a finite factorizable contribution as a result of symmetry-breaking effect. At any rate, the predicted branching ratios for the charm-flavor-conserving decays $\Xi_c^0 \to \Lambda_c^+ \pi^-$ and $\Xi_c^+ \to \Lambda_c^+ \pi^0$ are of order $10^{-3} \sim 10^{-4}$ and should be readily accessible in the near future.

VII. SEMILEPTONIC DECAYS

The exclusive semileptonic decays of charmed baryons: $\Lambda_c^+ \to \Lambda e^+(\mu^+)\nu_e$, $\Xi_c^+ \to \Xi^0 e^+ \nu_e$ and $\Xi_c^0 \to \Xi^- e^+ \nu_e$ have been observed experimentally. Their rates depend on the $\mathcal{B}_c \to \mathcal{B}$ form factors $f_i(q^2)$ and $g_i(q^2)$ $(i = 1, 2, 3)$ defined in Eq. (6.4). These form factors have been evaluated in the non-relativistic quark model [81, 85, 86], the MIT bag model [85], the relativistic quark model [87], the light-front quark model [88] and QCD sum rules [89, 90]. Experimentally, the only information available so far is the form-factor ratio measured in the semileptonic decay $\Lambda_c \to \Lambda e \bar{\nu}$. In the heavy quark limit, the six $\Lambda_c \to \Lambda$ form factors are reduced to two:

$$\langle \Lambda(p) | \bar{s}\gamma_\mu(1-\gamma_5)c | \Lambda_c(v) \rangle = \bar{u}_{_\Lambda} \left(F_1^{\Lambda_c\Lambda}(v \cdot p) + \not{v} F_2^{\Lambda_c\Lambda}(v \cdot p) \right) \gamma_\mu (1-\gamma_5) u_{_{\Lambda_c}}. \tag{7.1}$$

Assuming a dipole q^2 behavior for form factors, the ratio $R = \tilde{F}_2^{\Lambda_c\Lambda}/\tilde{F}_1^{\Lambda_c\Lambda}$ is measured by CLEO to be [91]

$$R = -0.31 \pm 0.05 \pm 0.04 \,. \tag{7.2}$$

Various model predictions of the charmed baryon semileptonic decay rates and decay asymmetries are shown in Table X. Dipole q^2 dependence for form factors is assumed whenever the form factor momentum dependence is not available in the model. The predicted rates cited from [85] include QCD corrections. However, as stressed in [1], it seems that QCD effects computed in [85] are unrealistically too large. Moreover, the calculated heavy-heavy baryon form factors in [85] at zero recoil do not satisfy the constraints imposed by heavy quark symmetry [81]. From Table X we see that the computed branching ratios of $\Lambda_c^+ \to \Lambda e^+\nu$ lie in the range $1.4\% \sim 2.6\%$, in agreement with experiment, $(2.1 \pm 0.6)\%$ [3]. Branching ratios of $\Xi_c^0 \to \Xi^- e^+\nu$ and $\Xi_c^+ \to \Xi^0 e^+\nu$ are predicted to fall into the ranges $(0.8 \sim 2.0)\%$ and $(3.3 \sim 8.1)\%$, resepctively. Experimentally, only the ratios of the branching fractions are available so far [3]

$$\frac{\Gamma(\Xi_c^+ \to \Xi^0 e^+\nu)}{\Gamma(\Xi_c^+ \to \Xi^-\pi^+\pi^+)} = 2.3 \pm 0.6^{+0.3}_{-0.6}, \qquad \frac{\Gamma(\Xi_c^0 \to \Xi^- e^+\nu)}{\Gamma(\Xi_c^0 \to \Xi^-\pi^+)} = 3.1 \pm 1.0^{+0.3}_{-0.5} \,. \tag{7.3}$$

VIII. ELECTROMAGNETIC AND WEAK RADIATIVE DECAYS

Although radiative decays are well measured in the charmed meson sector, e.g. $D^* \to D\gamma$ and $D_s^+ \to D_s^+\gamma$, only three of the radiative modes in the charmed baryon sector have been seen, namely, $\Xi_c'^0 \to \Xi_c^0\gamma$, $\Xi_c'^+ \to \Xi_c^+\gamma$ and $\Omega_c^{*0} \to \Omega_c^0\gamma$. This is understandable because $m_{\Xi_c'} - m_{\Xi_c} \approx 107$ MeV and $m_{\Omega_c^*} - m_{\Omega_c} \approx 71$ MeV. Hence, Ξ_c' and Ω_c^* are governed by the electromagnetic decays. However, it will be difficult to measure the rates of these decays because these states are too narrow to be experimentally resolvable. Nevertheless, we shall systematically study the two-body electromagnetic decays of charmed baryons and also weak radiative decays.

A. Electromagnetic decays

In the baryon sector, the following two-body electromagnetic decays are of interest:

$$\begin{aligned}
B_6 \to B_{\bar{3}} + \gamma \;&:\; \Sigma_c \to \Lambda_c + \gamma, \quad \Xi_c' \to \Xi_c + \gamma, \\
B_6^* \to B_{\bar{3}} + \gamma \;&:\; \Sigma_c^* \to \Lambda_c + \gamma, \quad \Xi_c'^* \to \Xi_c + \gamma, \\
B_6^* \to B_6 + \gamma \;&:\; \Sigma_c^* \to \Sigma_c + \gamma, \quad \Xi_c'^* \to \Xi_c' + \gamma, \quad \Omega_c^* \to \Omega_c + \gamma,
\end{aligned} \tag{8.1}$$

where we have denoted the spin $\frac{1}{2}$ baryons as B_6 and $B_{\bar{3}}$ for a symmetric sextet $\mathbf{6}$ and antisymmetric antitriplet $\bar{\mathbf{3}}$, respectively, and the spin $\frac{3}{2}$ baryon by B_6^*.

An ideal theoretical framework for studying the above-mentioned electromagnetic decays is provided by the formalism in which the heavy quark symmetry and the chiral symmetry of light quarks are combined [21, 22]. When supplemented by the nonrelativistic quark model, the formalism determines completely the low energy dynamics of heavy hadrons. The electromagnetic interactions of heavy hadrons consist of two distinct contributions: one from gauging electromagnetically the chirally invariant strong interaction Lagrangians for heavy mesons and baryons given in [21, 22], and the other from the anomalous magnetic moment couplings of the heavy particles. The heavy

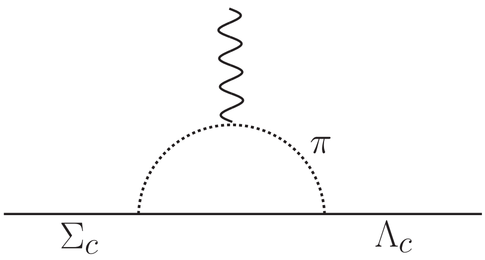

FIG. 5: Chiral loop contribution to the E2 amplitude of $\Sigma_c^* \to \Lambda_c\gamma$.

quark symmetry reduces the number of free parameters needed to describe the magnetic couplings to the photon. There are two undetermined parameters for the ground state heavy baryons. All these parameters are related simply to the magnetic moments of the light quarks in the nonrelativistic quark model. However, the charmed quark is not particularly heavy ($m_c \simeq 1.6$ GeV), and it carries a charge of $\frac{2}{3}e$. Consequently, the contribution from its magnetic moment cannot be neglected. The chiral and electromagnetic gauge-invariant Lagrangian for heavy baryons can be found in Eqs. (3.8) and (3.9) of [92].

The amplitudes of electromagnetic decays are given by [92]

$$\begin{aligned} A(B_6 \to B_{\bar{3}} + \gamma) &= i\eta_1 \bar{u}_{\bar{3}} \sigma_{\mu\nu} k^\mu \varepsilon^\nu u_6, \\ A(B_6^* \to B_{\bar{3}} + \gamma) &= i\eta_2 \epsilon_{\mu\nu\alpha\beta} \bar{u}_{\bar{3}} \gamma^\nu k^\alpha \varepsilon^\beta u^\mu, \\ A(B_6^* \to B_6 + \gamma) &= i\eta_3 \epsilon_{\mu\nu\alpha\beta} \bar{u}_6 \gamma^\nu k^\alpha \varepsilon^\beta u^\mu, \end{aligned} \tag{8.2}$$

where k_μ is the photon 4-momentum and ε_μ is the polarization 4-vector. The corresponding decay rates are [92]

$$\begin{aligned} \Gamma(B_6 \to B_{\bar{3}} + \gamma) &= \eta_1^2 \frac{k^3}{\pi}, \\ \Gamma(B_6^* \to B_{\bar{3}} + \gamma) &= \eta_2^2 \frac{k^3}{3\pi} \frac{3m_i^2 + m_f^2}{4m_i^2}, \\ \Gamma(B_6^* \to B_6 + \gamma) &= \eta_3^2 \frac{k^3}{3\pi} \frac{3m_i^2 + m_f^2}{4m_i^2}, \end{aligned} \tag{8.3}$$

where m_i (m_f) is the mass of the parent (daughter) baryon.

The coupling constants η_i can be calculated using the quark model [92]; some of them are

$$\begin{aligned} \eta_1(\Sigma_c^+ \to \Lambda_c^+) &= \frac{e}{6\sqrt{3}}\left(\frac{2}{M_u} + \frac{1}{M_d}\right), \quad \eta_2(\Sigma_c^{*+} \to \Lambda_c^+) = \frac{e}{3\sqrt{6}}\left(\frac{2}{M_u} + \frac{1}{M_d}\right), \\ \eta_3(\Sigma_c^{*++} \to \Sigma_c^{++}) &= \frac{2\sqrt{2}e}{9}\left(\frac{1}{M_u} - \frac{1}{M_c}\right), \quad \eta_3(\Sigma_c^{*0} \to \Sigma_c^0) = \frac{2\sqrt{2}e}{9}\left(-\frac{1}{2M_d} - \frac{1}{M_c}\right), \\ \eta_3(\Sigma_c^{*+} \to \Sigma_c^+) &= \frac{\sqrt{2}e}{9}\left(\frac{1}{M_u} - \frac{1}{2M_d} - \frac{2}{M_c}\right), \quad \eta_3(\Xi_c^{\prime *+} \to \Xi_c^+) = \frac{e}{3\sqrt{6}}\left(\frac{2}{M_u} + \frac{1}{M_s}\right), \\ \eta_3(\Xi_c^{\prime *0} \to \Xi_c^0) &= \frac{e}{3\sqrt{6}}\left(-\frac{1}{M_d} + \frac{1}{M_s}\right), \quad \eta_3(\Omega_c^{*0} \to \Omega_c^0) = \frac{2\sqrt{2}e}{9}\left(-\frac{1}{2M_s} - \frac{1}{M_c}\right). \end{aligned} \tag{8.4}$$

Using the constituent quark masses, $M_u = 338$ MeV, $M_d = 322$ MeV and $M_s = 510$ MeV [3], the calculated results are summarized in the second column of Table XI. A similar procedure is followed

TABLE XI: Electromagnetic decay rates (in units of keV) of charmed baryons.

Decay	HHChPT +QM [23, 92]	Ivanov et al. [87]	Bañuls et al. [93]	Tawfiq et al. [94]	Experiment [3]
$\Sigma_c^+ \to \Lambda_c^+\gamma$	88	60.7 ± 1.5		87	
$\Sigma_c^{*++} \to \Sigma_c^{++}\gamma$	1.4			3.04	
$\Sigma_c^{*+} \to \Sigma_c^+\gamma$	0.002	0.14 ± 0.004		0.19	
$\Sigma_c^{*+} \to \Lambda_c^+\gamma$	147	151 ± 4			
$\Sigma_c^{*0} \to \Lambda_c^0\gamma$	1.2			0.76	
$\Xi_c'^+ \to \Xi_c^+\gamma$	16	12.7 ± 1.5			seen
$\Xi_c'^0 \to \Xi_c^0\gamma$	0.3	0.17 ± 0.02	1.2 ± 0.7		seen
$\Xi_c'^{*+} \to \Xi_c^+\gamma$	54	54 ± 3			
$\Xi_c'^{*0} \to \Xi_c^0\gamma$	1.1	0.68 ± 0.04	5.1 ± 2.7		
$\Omega_c^{*0} \to \Omega_c^0\gamma$	0.9				seen

in [94] where the heavy quark symmetry is supplemented with light-diquark symmetries to calculate the widths of $\Sigma_c^+ \to \Lambda_c^+\gamma$ and $\Sigma_c^* \to \Sigma_c\gamma$. The authors of [87] apply the relativistic quark model to predict various electromagnetic decays of charmed baryons. Besides the magnetic dipole (M1) transition, the author of [95] also considered and estimated the electric quadrupole (E2) amplitude for $\Sigma_c^{*+} \to \Lambda_c^+\gamma$ arising from the chiral loop correction (Fig. 5). A detailed analysis of the E2 contributions was presented in [93]. The E2 amplitudes appear at different higher orders for the three kinds of decays: $\mathcal{O}(1/\Lambda_\chi^2)$ for $B_6^* \to B_6 + \gamma$, $\mathcal{O}(1/m_Q\Lambda_\chi^2)$ for $B_6^* \to B_{\bar{3}} + \gamma$ and $\mathcal{O}(1/m_Q^3\Lambda_\chi^2)$ for $B_6 \to B_{\bar{3}} + \gamma$. Therefore, the E2 contribution to $B_6 \to B_{\bar{3}} + \gamma$ is completely negligible.

Chiral-loop corrections to the M1 electromagnetic decays and to the strong decays of heavy baryons have been computed at the one loop order in [96]. The leading chiral-loop effects we found are nonanalytic in the forms of m/Λ_χ and $(m^2/\Lambda_\chi^2)\ln(\Lambda^2/m^2)$ (or $m_q^{1/2}$ and $m_q \ln m_q$, with m_q being the light quark mass). Some results are [96]

$$\Gamma(\Sigma_c^+ \to \Lambda_c^+\gamma) = 112\,\text{keV}, \quad \Gamma(\Xi_c'^+ \to \Xi_c^+\gamma) = 29\,\text{keV}, \quad \Gamma(\Xi_c'^0 \to \Xi_c^0\gamma) = 0.15\,\text{keV}, \tag{8.5}$$

which should be compared with the corresponding quark-model results: 88 keV, 16 keV and 0.3 keV (Table XI).

The electromagnetic decay $\Xi_c'^{*0} \to \Xi_c^0\gamma$ is of special interest. It has been advocated in [97] that a measurement of its branching ratio will determine one of the coupling constants in HHChPT, namely, g_1. The radiative decay $\Xi_c'^{*0} \to \Xi_c^0\gamma$ is forbidden at tree level in SU(3) limit [see Eq. (8.4)]. In heavy baryon chiral perturbation theory, this radiative decay is induced via chiral loops where SU(3) symmetry is broken by the light current quark masses. By identifying the chiral loop contribution to $\Xi_c'^{*0} \to \Xi_c^0\gamma$ with the quark model prediction given in Eq. (8.4), it was found in [23] that one of the two possible solutions is in accord with the quark model expectation for g_1.

For the electromagnetic deacys of p-wave charmed baryons, the search of $\Lambda_c(2593)^+ \to \Lambda_c^+\gamma$ and $\Lambda_c^+(2625)^+ \to \Lambda_c^+\gamma$ has been failed so far. On the theoretical side, the interested reader is referred to [32, 34, 87, 94, 97, 98] for more details.

The electromagnetic decays considered so far do not test critically the heavy quark symmetry

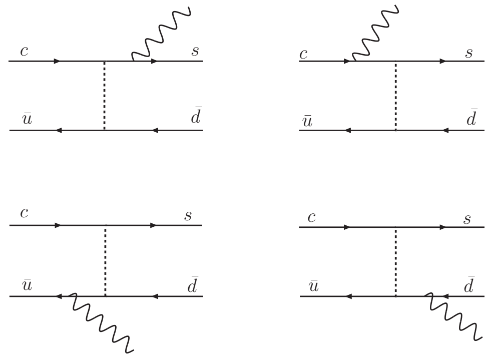

FIG. 6: W-exchange diagrams contributing to the quark-quark bremsstrahlung process $c + \bar{u} \to s + \bar{d} + \gamma$. The W-annihilation type diagrams are not shown here.

nor the chiral symmetry. The results follow simply from the quark model. There are examples in which both the heavy quark symmetry and the chiral symmetry enter in a crucial way. These are the radiative decays of heavy baryons involving an emitted pion. Some examples which are kinematically allowed are

$$\Sigma_c \to \Lambda_c \pi\gamma, \quad \Sigma_c^* \to \Lambda_c \pi\gamma, \quad \Sigma_c^* \to \Sigma_c \pi\gamma, \quad \Xi_c^* \to \Xi_c \pi\gamma. \tag{8.6}$$

For an analysis of the decay $\Sigma_c \to \Lambda_c \pi\gamma$, see [92].

B. Weak radiative decays

At the quark level, there are three different types of processes which can contribute to the weak radiative decays of heavy hadrons, namely, single-, two- and three-quark transitions [100]. The single-quark transition mechanism comes from the so-called electromagnetic penguin diagram. Unfortunately, the penguin process $c \to u\gamma$ is very suppressed and hence it plays no role in charmed hadron radiative decays. There are two contributions from the two-quark transitions: one from the W-exchange diagram accompanied by a photon emission from the external quark, and the other from the same W-exchange diagram but with a photon radiated from the W boson. The latter is typically suppressed by a factor of $m_q k/M_W^2$ (k being the photon energy) as compared to the former bremsstrahlung process [99]. For charmed baryons, the Cabibbo-allowed decay modes via $c\bar{u} \to s\bar{d}\gamma$ (Fig. 6) or $cd \to us\gamma$ are

$$\Lambda_c^+ \to \Sigma^+\gamma, \qquad \Xi_c^0 \to \Xi^0\gamma. \tag{8.7}$$

Finally, the three-quark transition involving W-exchange between two quarks and a photon emission by the third quark is quite suppressed because of very small probability of finding three quarks in adequate kinematic matching with the baryons [100, 101].

The general amplitude of the weak radiative baryon decay reads

$$A(\mathcal{B}_i \to \mathcal{B}_f\gamma) = i\bar{u}_f(a + b\gamma_5)\sigma_{\mu\nu}\varepsilon^\mu k^\nu u_i, \tag{8.8}$$

where a and b are parity-conserving and -violating amplitudes, respectively. The corresponding decay rate is

$$\Gamma(\mathcal{B}_i \to \mathcal{B}_f\gamma) = \frac{1}{8\pi}\left(\frac{m_i^2 - m_f^2}{m_i}\right)^3 (|a|^2 + |b|^2). \tag{8.9}$$

Nonpenguin weak radiative decays of charmed baryons such as those in (8.7) are characterized by emission of a hard photon and the presence of a highly virtual intermediate quark between the electromagnetic and weak vertices. It has been shown in [102] that these features should make possible to analyze these processes by perturbative QCD; that is, these processes are describable by an effective local and gauge invariant Lagrangian:

$$\mathcal{H}_{\rm eff}(c\bar{u}\to s\bar{d}\gamma) = \frac{G_F}{2\sqrt{2}}V_{cs}V_{ud}^*(c_+O_+^F + c_-O_-^F), \tag{8.10}$$

with

$$O_\pm^F(c\bar{u}\to s\bar{d}\gamma) = \frac{e}{m_i^2-m_f^2}\left\{\left(e_s\frac{m_f}{m_s}+e_u\frac{m_i}{m_u}\right)\left(\tilde{F}_{\mu\nu}+iF_{\mu\nu}\right)O_\pm^{\mu\nu}\right. \tag{8.11}$$

$$\left. - \left(e_d\frac{m_f}{m_d}+e_c\frac{m_i}{m_c}\right)\left(\tilde{F}_{\mu\nu}-iF_{\mu\nu}\right)O_\mp^{\mu\nu}\right\}, \tag{8.12}$$

where $m_i = m_c + m_u$, $m_f = m_s + m_d$, $\tilde{F}_{\mu\nu} \equiv \frac{1}{2}\epsilon_{\mu\nu\alpha\beta}F^{\alpha\beta}$ and

$$O_\pm^{\mu\nu} = \bar{s}\gamma^\mu(1-\gamma_5)c\bar{u}\gamma^\nu(1-\gamma_5)d \pm \bar{s}\gamma^\mu(1-\gamma_5)d\bar{u}\gamma^\nu(1-\gamma_5)c. \tag{8.13}$$

For the charmed baryon radiative decays, one needs to evaluate the matrix element $\langle\mathcal{B}_f|O_\pm^{\mu\nu}|\mathcal{B}_i\rangle$. Since the quark-model wave functions best resemble the hadronic states in the frame where both baryons are static, the static MIT bag model [71] was thus adopted in [102] for the calculation. The predictions are

$$\begin{aligned}\mathcal{B}(\Lambda_c^+\to\Sigma^+\gamma) &= 4.9\times10^{-5}, \qquad \alpha(\Lambda_c^+\to\Sigma^+\gamma) = -0.86\,,\\ \mathcal{B}(\Xi_c^0\to\Xi^0\gamma) &= 3.6\times10^{-5}, \qquad \alpha(\Xi_c^0\to\Xi^0\gamma) = -0.86\,.\end{aligned} \tag{8.14}$$

A different analysis of the same decays was carried out in [103] with the results

$$\begin{aligned}\mathcal{B}(\Lambda_c^+\to\Sigma^+\gamma) &= 2.8\times10^{-4}, \qquad \alpha(\Lambda_c^+\to\Sigma^+\gamma) = 0.02\,,\\ \mathcal{B}(\Xi_c^0\to\Xi^0\gamma) &= 1.5\times10^{-4}, \qquad \alpha(\Xi_c^0\to\Xi^0\gamma) = -0.01\,.\end{aligned} \tag{8.15}$$

Evidently, these predictions (especially the decay asymmetry) are very different from the ones obtained in [102].

Finally, it is worth remarking that, in analog to the heavy-flavor-conserving nonleptonic weak decays as discussed in Sec. VI, there is a special class of weak radiative decays in which heavy flavor is conserved. Some examples are $\Xi_c\to\Lambda_c\gamma$ and $\Omega_c\to\Xi_c\gamma$. In these decays, weak radiative transitions arise from the diquark sector of the heavy baryon whereas the heavy quark behaves as a spectator. However, the dynamics of these radiative decays is more complicated than their counterpart in nonleptonic weak decays, e.g., $\Xi_c\to\Lambda_c\pi$. In any event, it deserves a detailed study.

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