Chapter 30

CP and T Violation

The violation of the $CP$ symmetry, where $C$ and $P$ are the charge-conjugation and parity-transformation operators, respectively, is one of the fundamental and most exciting phenomena in particle physics. Although weak interactions are not invariant under $P$ (and $C$) transformations, as discovered in 1957, it was believed for several years that the product $CP$ was preserved. However, in 1964, it was discovered through the observation of $K_L \to \pi^+ \pi^-$ decays that weak interactions are not invariant under $CP$ transformations [262]. After this discovery, many observations show us that $CP$ violation has been established in both $K$ and $B$ systems [263]. All these measurements are consistent with the Kobayashi-Maskawa (KM) picture of $CP$ violation.

However, people still believe that there must be new sources of $CP$ violation beyond the SM prediction since:

- **The baryon asymmetry of the Universe:**
  Baryogenesis is a consequence $CP$ violating processes [264]. Therefore the present baryon number, which is accurately deduced from nucleosynthesis and CMBR constraints,

\[
Y_B \equiv \frac{n_B - n_{\bar{B}}}{s} \approx 9 \times 10^{-11},
\]

is essentially a $CP$ violating observable. The surprising point is that the KM mechanism for $CP$ violation fails to account for it [263].

- **Non-vanishing neutrino masses:**
  It is also interesting to note that the evidence for non-vanishing neutrino masses that we obtained over the last few years points towards an origin beyond the SM [266], raising the question of having $CP$ violation in the neutrino sector, which could be studied, in the more distant future, at dedicated neutrino factory [267].

It is very interesting to look for $CP$ violation in the $D$ system, most factors favour dedicated searched for $CP$ violation in Charm transitions:

- New physics should be somewhere in the corner, since baryogenesis implies the existence of New Physics (NP) in $CP$ violation dynamics. It will be of interest

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to undertake dedicated searches for $CP$ asymmetries in Charm decays, where the SM predicts very small effects smaller than $O(10^{-3})$, and it can arise only in *singly Cabibbo-suppressed* (SCS) transitions. Significant larger values would signal NP. Any asymmetry in CF and DCS decays requires the intervention of NP (except for $D^{\pm} \to K_{S}\pi^{\pm}$ [1], where the $CP$ impurity in $K_{S}$ induces an asymmetry of $3.3 \times 10^{-3}$).

- Secondly, the neutral $D$ system is the only one where the external up-sector quarks are involved. Thus it probes models in which the up-sector plays a special role, such as supersymmetric models with alignment [265, 268] and, more generally, models in which CKM mixing is generated in the up sector.

- Third, SCS decays are sensitive to new physics contributions to penguin and dipole operator. As concern this point, among all hadronic $D$ decays, the SCS decays ($c \to uq\bar{q}$) are uniquely sensitive to new contributions to the $\Delta C = 1$ QCD penguin and chromomagnetic dipole operator [269, 270]. In particular, such contributions can affect neither CF ($c \to s\bar{d}u$) nor the DCS ($c \to d\bar{s}u$) decays.

- There are rich light resonances in $D$ mass region, and the final state interaction is large and enlarge the strong phase shift which required for the direct $CP$ violation.

- Decays to final states of more than two pseudoscalar or one pseudoscalar and one vector meson contain more dynamical information than given by their widths. The Dalitz plot analysis can exhibit $CP$ asymmetries that might be considerably larger than those for the width [271].

$CP$ asymmetries in integrated partial widths depend on hadronic matrix elements and (strong) phase shifts, neither of which can be predicted accurately. However the craft of theoretical engineering can be practiced with profit here. One makes an ansatz for the general form of the matrix elements and phase shifts that are included in the description of $D \to PP, PV, VV$ etc. channels, where $P$ and $V$ denote pseudoscalar and vector mesons, and fits them to the measured branching ratios on the Cabibbo allowed, once and twice forbidden level. If one has sufficiently accurate and comprehensive data, one can use these fitted values of the hadronic parameters to predict $CP$ asymmetries. Such analyses have been undertaken in the past [272], but the data base was not as broad and precise as one would like [271]. *CLEO* and *BESIII* measurements will certainly lift such studies to a new level of reliability.

There are several ways to study $CP$ violation in charm decays [280]. We can look for direct $CP$ violation, even in charged decays; we can look for $CP$ violation via mixing; $T$ violation can be examined in 4-body $D$ meson decays, assuming $CPT$ conservation, by measuring triple-product correlations [281]. Finally, the quantum coherence present in correlated $D^{0}\bar{D}^{0}$ decays of the $\psi(3770)$ can be exploited [282]. In table 30.1, we summarize the recent measurements of $CP$ violating asymmetries, $A_{CP}$ (or $A_{T}$) in Charm decays. BaBar compares $D^{+}$ versus $D^{-}$ rates. The CLEO results is obtained using a Dalitz plot analysis. CDF uses the decay of $D^{*+} \to D^{0}\pi_{soft}^{+}$ as a flavor tag, and FOCUS uses triple product correlations to measure $T$ violation.
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Table 30.1: Measurements of $CP$ violating asymmetries, $A_{CP}$ (or $A_T$) in Charm decays. BaBar compares $D^+$ versus $D^-$ rates. The CLEO results is obtained using a Dalitz plot analysis. CDF uses the decay of $D^{*+} \to D^0 \pi^+$ as a flavor tag, and FOCUS uses triple product correlations to measure $T$ violation.

In order to discuss the $CP$ violation in neutral $D$ system, we use the following notations as described in the previous sections:

$$
\tau \equiv \Gamma_D t, \quad \Gamma_D \equiv \frac{\Gamma_{D_H} + \Gamma_{D_L}}{2},
$$

$$
A_f \equiv A(D^0 \to f), \quad \bar{A}_f \equiv A(D^0 \to \bar{f}),
$$

$$
A_T \equiv A(D^0 \to \bar{T}), \quad \bar{A}_T \equiv A(D^0 \to T),
$$

$$
x \equiv \frac{\Delta m_D}{\Gamma_D} = \frac{m_{D_H} - m_{D_L}}{\Gamma_D}, \quad y \equiv \frac{\Delta \Gamma_D}{2\Gamma_D} = \frac{\Gamma_{D_H} - \Gamma_{D_L}}{2\Gamma_D},
$$

$$
\lambda_f \equiv \frac{q}{p} \frac{\bar{A}_f}{A_f}, \quad A_M \equiv \left|\frac{q}{p}\right| - 1, \quad R_f \equiv \left|\frac{\bar{A}_f}{A_f}\right|,
$$

(30.2)

where $D_H$ and $D_L$ stand for the heavy and light mass eigenstates, and $q$ and $p$ are defined as in Eq. (29.4). We distinguish three types of $CP$-violation effects in meson decays:

- **$CP$ violation in decay** is defined by

  $$
  \left|\frac{\bar{A}_T}{A_f}\right| \neq 1.
  $$

  (30.3)

In the charged $D$ decays with mixing effects, it is the only possible source of $CP$ asymmetries:

- **$CP$ violation in mixing** is defined as

  $$
  \left|\frac{q}{p}\right| \neq 1.
  $$

  (30.4)
In the charged-current semileptonic neutral $D$ decays, $|A_{l^+X}| = |A_{l^-X}|$ and $A_{l^-X} = \overline{A}_{l^+X} = 0$, as in the SM or most of the reasonable extensions of SM, this is the only source of $CP$ violation, and can be measured in the asymmetry of "wrong-sign" decays induced by oscillations.

- $CP$-violation in the interference between a decay without mixing, $D^0 \rightarrow f$, and a decay with mixing, $D^0 \rightarrow \overline{D}^0 \rightarrow f$, and is defined by

$$\Im \left( \frac{q}{p} A_f \right) \neq 0. \quad (30.5)$$

This form of $CP$ violation can be observed in the asymmetry of $D^0$ and $\overline{D}^0$ decays into common final states, such as $CP$ eigenstates $f_{CP}$.

Example of the three types of $CP$ violation will be given in the following sections.

### 30.1 $CP$ Violation in $D^0 - \overline{D}^0$ Mixing

This kind of asymmetry can be best isolated in the semileptonic decay of the neutral $D$ mesons, as discussed in Ref. [283], in the case of $D^0$ meson, it can be measured in the asymmetry of "wrong-sign" decays ($A_{SL}$):

$$A_{SL} \equiv \frac{\Gamma(D^0(t)_{phys} \rightarrow l^+X) - \Gamma(D^0(t)_{phys} \rightarrow l^-X)}{\Gamma(D^0(t)_{phys} \rightarrow l^+X) + \Gamma(D^0(t)_{phys} \rightarrow l^-X)}$$

$$= \frac{1 - |q/p|^4}{1 + |q/p|^4}. \quad (30.6)$$

Here $D^0(0)_{phys} = D^0$ and $D^0(0)_{phys} = \overline{D}^0$. Note that the final states in Eq. (30.6) contains "wrong charged" leptons and can only be reached in the presence of $D^0 - \overline{D}^0$ oscillations. This one studies effectively the difference between the rates for $\overline{D}^0 \rightarrow D^0 \rightarrow l^+X$ and $D^0 \rightarrow \overline{D}^0 \rightarrow l^-X$. As the phases in the transitions $D^0 \rightarrow D^0$ and $D^0 \rightarrow \overline{D}^0$ differ each other, a non-vanishing $CP$ violation follows. This asymmetry is expected to be tiny in both the SM and many of its extensions. There no any precise predictions since the large hadronic uncertainties are involved in the calculation of the mixing rate in neutral $D$ system.

At $\psi(3770)$ peak, this kind of $CP$-violating signal can manifest itself in the like-sign dilepton events of ($D^0\overline{D}^0$) pairs:

$$A_{SL} \equiv \frac{R(l^+_1 X, l^+_2 X) - R(l^-_1 X, l^-_2 X)}{R(l^+_1 X, l^+_2 X) + R(l^-_1 X, l^-_2 X)} = \frac{1 - |q/p|^4}{1 + |q/p|^4}, \quad (30.7)$$

where $R(l^+_1 X, l^+_2 X)$ and $R(l^-_1 X, l^-_2 X)$ are the production rates for the like-sign dilepton on $\psi(3770)$ defined in Ref. [284]. Note that this asymmetry is not only independent of the time distributions, but also independent of the charge-conjugation parity $C$ of the ($D^0\overline{D}^0$) pairs, thus it can be measured by using the time-integrated dilepton events at either $\psi(3770)$ or $\psi(4170)$ resonances.
30.2 CP Violation in Decay

This type of CP violation (also called "direct CP violation") occurs when the absolute value of the decay amplitude $A_f$ for $D$ decaying to a final state $f$ is different from the one of corresponding CP-conjugated amplitude, i.e. $|A_f| \neq |\overline{A}_f|$. This type of CP violation occurs in both charged and neutral $D$ decays and induced by $\Delta C = 1$ effective operators.

In the charged $D$ meson decays, where mixing effect are absent, this is the only possible source of CP asymmetries:

$$A^{CP}_{f^\pm} = \frac{\Gamma(D^- \rightarrow f^-) - \Gamma(D^+ \rightarrow f^+)}{\Gamma(D^- \rightarrow f^-) + \Gamma(D^+ \rightarrow f^+)} = \frac{|\overline{A}_f - A_f|^2 - 1}{|\overline{A}_f - A_f|^2 + 1}, \quad (30.8)$$

where $\Gamma_{f^\pm}$ represents the $D^\pm \rightarrow f^\pm$ decay rate. A two-component decay amplitude with weak and strong phase differences is required for this type of CP violation. If, for example, there are two such contributions, $A_f = a_1 + a_2$, we have

$$A_f = |a_1|e^{i\phi_f}[1 + r_f e^{i(\Delta_f + \theta_f)}],$$

$$\overline{A}_f = |a_1|e^{-i\phi_f}[1 + r_f e^{i(\Delta_f - \theta_f)}], \quad (30.9)$$

where $\Delta_f$ corresponds to the strong phase difference and $\theta_f$ corresponds to the weak phase difference between the CP-conserving ($a_1$ from tree level contribution in the SM) and CP-violating parts of the decay amplitude and $r_f$ represents the small ratio, $r_f = |a_2|/|a_1|$, $\phi_f$ is the weak phase from the SM tree level contribution.

It is now straightforward to evaluate the CP asymmetry in the charged $D$ decays as:

$$A^{CP}_{f^\pm} = -\frac{2|r_f|\sin(\Delta_f)\sin(\theta_f)}{1 + |r_f|^2 + 2|r_f|\cos(\Delta_f)\cos(\theta_f)}. \quad (30.10)$$

No reliable model-independent predictions exist for the $\Delta_f$, it is believed that it could be quite large due to the abundance of light-quark resonances in the vicinity of the $D$-meson mass inducing large final-state interaction (FSI) phases. The quantity of most interest to theory is the weak phase difference $\theta_f$. Its extraction from the asymmetry requires, however, that the amplitude ratio $r_f$ and the strong phase difference $\Delta_f$ are known. Both quantities are difficult to calculate due to non-perturbative hadronic parameters. In the SM, relative weak phases can only be obtained in SCS decays, for instance, via the interference between spectator and penguin amplitudes. As the most optimistic model-dependent estimates put the SM predictions for the asymmetry $A^{CP} < 0.1\%$ [285], an observation of any CP-violating signal in the current round of experiments will a sign of new physics.

Specific model calculations [286] for $D \rightarrow K\bar{K}, \pi\pi, K^*\bar{K}$, three-body modes, etc. yield this order of magnitude for the effect. New physics could enter, for example, through large phases in penguin diagram. This could give asymmetries of the order of 1% or larger. On the other hand, CF decays do not have two amplitudes with different weak phases, and therefore the $CP$ asymmetry is zero in the SM. Some new physics scenarios may provide extra phases and could give asymmetries as large as 1%.
30.3 \textit{CP} Violation in the Interference of Decays with and without Mixing

This type of \textit{CP} violation is possible for common final states to which both $D^0$ and $\bar{D}^0$ can decay. It is usually associated with the relative phase between mixing and decay contributions as described in Eq. (30.5). It can be studied in both time-dependent and time-integrated asymmetries.

(1) \textit{CP} eigenstate

In general, for a \textit{CP} even (old) eigenstates, the decay amplitudes can be written as

$$A_f = |a_1|e^{i\phi_T}[1 + r_f e^{i(\Delta_f + \theta_f)}],$$

$$\overline{A}_f = \eta_f^{CP} \times \overline{A}_f = \eta_f^{CP}[a_1 |e^{-i\phi_T}[1 + r_f e^{i(\Delta_f - \theta_f)}]],$$

where $\eta_f^{CP} = (+)$ for \textit{CP} even (odd) states and we have used $\text{CP}|D^0\rangle = -|\bar{D}^0\rangle$. Neglecting $r_f$ in Eq. (30.11), $\lambda_f$ can be written as

$$\lambda_f = -\eta_f^{CP} R_m e^{i\phi},$$

$$\overline{\lambda}_f = -\eta_f^{CP} R^{-1}_m e^{-i\phi},$$

where $\phi$ is the relative weak phase between the mixing amplitude and the decay amplitude. The time-integrated \textit{CP} asymmetry for a final state \textit{CP} eigenstate $f$ is defined as follows:

$$A_f^{CP} \equiv \frac{\Gamma(D^0 \to f) - \Gamma(\bar{D}^0 \to f)}{\Gamma(D^0 \to f) - \Gamma(\bar{D}^0 \to f)}$$

(30.13)

Given experimental constraints, one can take $x, y, r_f \ll 1$ and expand to the leading order in these parameters, and get [269]:

$$A_f^{CP} = 2r_f \sin \Delta_f \sin \theta_f - \eta_f^{CP} \frac{y}{2}(R_m - R^{-1}_m) \cos \phi + \eta_f^{CP} \frac{x}{2}(R_m + R^{-1}_m) \sin \phi,$$

(30.14)

where the first term represents \textit{CP} violation in the decay, the second term is related to \textit{CP} violation in mixing, and the third term is for \textit{CP} violation in the interference between mixing and decay amplitudes. By neglecting the \textit{CP} violation in mixing and decay, one can get:

$$A_f^{CP} \approx \eta_f^{CP} x \sin \phi.$$ 

(30.15)

The above discussion is only for incoherent $D^0\bar{D}^0$ decays. In the case of $D^0\bar{D}^0$ produced coherently at BESIII, the $D^0\bar{D}^0$ pair system is in a state with charge parity $C = \eta$, which can be defined as [270, 284]

$$|D^0\bar{D}^0\rangle^C = \eta = \frac{1}{\sqrt{2}} \left[ |D^0\rangle |\bar{D}^0\rangle + \eta |D^0\rangle |\bar{D}^0\rangle \right],$$

(30.16)
where \( \eta \) is the charge conjugation parity or orbital angular momentum of the \( D^0 \bar{D}^0 \) pair. Thus, it is easy to see that \( D^0 \bar{D}^0 \) occur in a \( P \)-wave \((l = 1)\) in the reaction:

\[
\begin{align*}
e^+e^- & \to \gamma^* \to D^0 \bar{D}^0, \\
e^+e^- & \to \gamma^* \to D^0 \bar{D}^0, D^{*0} \bar{D}^0 \to D^0 \bar{D}^0 \pi^0, \\
e^+e^- & \to \gamma^* \to D^{*0} \bar{D}^0 \to D^0 \bar{D}^0 \pi^0 \pi^0,
\end{align*}
\]

and in an \( S \)-wave \((l=0)\) in the reactions:

\[
\begin{align*}
e^+e^- & \to \gamma^* \to D^0 \bar{D}^0, D^{*0} \bar{D}^0 \to D^0 \bar{D}^0 \gamma, \\
e^+e^- & \to \gamma^* \to D^{*0} \bar{D}^0 \to D^0 \bar{D}^0 \gamma \pi^0.
\end{align*}
\]

One can use the semileptonic decay of one \( D \) meson to tag the other \( D \) decaying to a \( CP \) eigenstate \( f \). We define the leptonic-tagging \( CP \) asymmetry \( A_{f \bar{f}}^{CP} \) as

\[
A_{f \bar{f}}^{CP} = \frac{R(l^- X, f) - R(l^+ X, f)}{R(l^- X, f) + R(l^+ X, f)} = \frac{N(l^- X, f) - N(l^+ X, f)}{N(l^- X, f) + N(l^+ X, f)},
\]

where \( R(l^- X, f) \) and \( R(l^+ X, f) \) are the time integrated decay rates of \( |D^0 \bar{D}^0\rangle_C^\eta \) into \((l^- X, f)\) and \((l^+ X, f)\) final state, respectively, and are defined as [270]:

\[
R(l^- X, f) = \int_0^\infty dt_1 \int_0^1 dt_2 \langle (l^- X, f)|\mathcal{H}|D^0 \bar{D}^0\rangle_C^\eta |^2,
\]

\[
R(l^+ X, f) = \int_0^\infty dt_1 \int_0^1 dt_2 \langle (l^+ X, f)|\mathcal{H}|D^0 \bar{D}^0\rangle_C^\eta |^2,
\]

are proportional to the number of events, \( N(l^- X, f) \) and \( N(l^+ X, f) \), for the semileptonic-tagged \( D^0 \bar{D}^0 \to f \). After a complicated calculation, in Ref. [270], D.S. Du got:

\[
A_{f \bar{f}}^{CP} = (1 + \eta) \eta_{f \bar{f}}^{CP} \left[ -\frac{y}{2} (R_m - R_m^{-1}) \cos \phi + \frac{x}{2} (R_m + R_m^{-1}) \sin \phi \right].
\]

Comparing Eq. (30.22) with Eq. (30.14) and neglecting \( CP \) violation in decay (namely, the first term in Eq. (30.14) is zero), Du finds that \( A_{f \bar{f}}^{CP} \) is just twice as large as \( A_{f \bar{f}}^{CP} \) when charge conjugation parity or the orbital angular momentum \( l \) is even. In table 30.2, we present an estimate of the number of all semi-leptonic tagged events of the type \( K(\pi) e \nu \) and \( K(\pi) \mu \nu \), where the neutrino is the only missing particle, with \( CP \)-eigenstates that can be collected in a \( 10^7 \) seconds (about 1 year) of running at \( \sqrt{s} = 4.17 \) GeV with luminosity of \( \mathcal{L} = 10^{33} \) cm\(^{-2}\) sec\(^{-1}\). The chain of the reactions considered is \( e^+e^- \to \gamma^* \to D^0 \bar{D}^0, D^{*0} \bar{D}^0 \to D^0 \bar{D}^0 \gamma \) and \( (D^0 \bar{D}^0) \to (l^+ X, f) \). The production cross-section used here is about \( \sigma(e^+e^- \to D^0 \bar{D}^0) = 2.6 \) nb, and the branching ratio assumed for the decay \( D^{*0} \to D^0 \gamma \) is 38%. The \( CP \)-eigenstate branching ratios are taken from PDG2006. The efficiencies are estimated based on solid angle and PID criteria. All the number are normalized via the branching ratios and efficiencies to the estimate for the above decay chain. As the results indicate in table 30.2, there are about 10,000 events would be observed in a year’s running at BESIII. Consequently, the observed \( CP \) asymmetry, \( A_{f \bar{f}}^{CP} \), could be measured with accuracy of 1.2% in a year’s time at BESIII.

(2) Non-\( CP \) eigenstate
Table 30.2: Estimate of the number of fully reconstructed semileptonic tagged events with CP-eigenstates in a one year running time at BESIII.

### 30.4 Rate of the CP Violation in the Coherent $D^0\bar{D}^0$ Pair

Let us consider the reaction:

$$e^+ e^- \rightarrow \psi(3770) \rightarrow D^0\bar{D}^0 \rightarrow f_a f_b,$$  

(30.23)

where $f_a$ and $f_b$ represent CP eigenstates of the same CP parity, i.e.

$$CP|f_a\rangle = \eta_a |f_a\rangle,$$

$$CP|f_b\rangle = \eta_b |f_b\rangle,$$

$$\eta_a \eta_b = +.$$  

(30.24)

The process in Eq. (30.23) can proceed only in the presence of CP violation, because:

$$CP|\psi(3770)\rangle = +|\psi(3770)\rangle,$$  

(30.25)

whereas

$$CP|f_a f_b\rangle = \eta_a \eta_b (-1)^{l_a-1} |f_a f_b\rangle = - |f_a f_b\rangle.$$  

(30.26)

Thus $CP(\text{initial}) \neq CP(\text{final})$, and CP invariance is broken.

Note that all these arguments cannot be applied for $D$ decays into vector states $D \rightarrow V_1 V_2$. For such a decay mode, in contrast to $D \rightarrow PP, PV$, is described by more than one independent amplitude. Therefore, the reaction $e^+ e^- \rightarrow D^0\bar{D}^0 \rightarrow (K^+ \rho)(K^+ \rho)$ can occur even in the absence of mixing and CP violation since the two decays could be described by different combinations of decay amplitudes. Its analysis could actually yield some interesting information on final state interactions in these decay modes.
30.5 CP Violation on Dalitz Plots

Three-body decays typically proceed through intermediate resonant 2-body modes. The final state is the result of the interference of all the intermediate states. These intermediate resonances cause a non-uniform distribution of events in phase-space, which can be analysed with the Dalitz plot (DP) technique [287, 288]. The analysis of DP makes it possible to measure the amplitudes and the phases of the intermediate channels. From this, their relative branching fractions can be deduced and information on the resonances that contribute to the final state can be gained, along with new information on light meson spectroscopy. Most notably, it is possible to look for WS resonant submodes that proceed via mixing or DC suppression.

30.5.1 with Flavor Tag

30.5.2 with CP Tag

30.6 T Violation

There are another type of CP-violating effects which can also reveal the presence of new physics: Triple-product (TP) correlations [289]. These take the form \( \vec{v}_1 \cdot (\vec{v}_2 \times \vec{v}_3) \), where each \( \vec{v}_i \) is spin or momentum. Since \( T(\vec{v}_i) = \vec{v}_i \), these TP’s are odd under time reversal \( T \) and hence, by the CPT theorem, also constitute potential signals of CP violation. One can construct the non-zero TP by measuring the non-zero value of the asymmetry:

\[
A_T \equiv \frac{\Gamma(\vec{v}_1 \cdot (\vec{v}_2 \times \vec{v}_3) > 0) - \Gamma(\vec{v}_1 \cdot (\vec{v}_2 \times \vec{v}_3) < 0)}{\Gamma(\vec{v}_1 \cdot (\vec{v}_2 \times \vec{v}_3) > 0) + \Gamma(\vec{v}_1 \cdot (\vec{v}_2 \times \vec{v}_3) < 0)},
\]

(30.27)

where \( \Gamma \) is the decay rate for a given process. However, a non-vanishing value for \( A_T \) is not necessarily due to the \( T \) transformation. This is because, in addition to reversing spins and momenta, the time reversal symmetry \( T \) also exchanges the initial and final states. Thus, in a given decay, a non-zero TP is not necessarily a signal of \( T \) violation. In particular, TP correlation can be due to the final state interaction, even if there is no \( CP \) violation. That is one typically finds that

\[
A_T \propto \sin(\phi + \delta),
\]

(30.28)

where \( \phi \) is a weak, \( CP \)-violating phase and \( \delta \) is a strong phase. From this we see that if \( \delta \neq 0 \), a TP correlation will appear, even without \( CP \) violation. To perform a stringent test of \( CP \) invariance, one has to study also the \( CP \) conjugate reaction and to construct a \( \overline{A}_T \) in the \( CP \)-conjugate decay process. One get a \( T \)-violating asymmetry:

\[
A_T^{CP} = \frac{1}{2}(A_T + \overline{A}_T).
\]

(30.29)

This is a true \( T \)-violating signal in that it is non-zero only if \( \phi \neq 0 \).

In fact, TP asymmetries are similar to direct \( CP \) asymmetries in two way [290]: (i) they are both obtained by comparing a signal in a given decay with the corresponding
signal in the \(CP\)-transformed process, and (ii) they both need the interference between two different decay amplitudes. However, there is one important difference between the direct \(CP\) and TP asymmetries. The direct \(CP\) asymmetry can be written

\[
A_{\text{dir}}^{CP} \propto \sin \phi \sin \delta, \tag{30.30}
\]

while, for the true \(T\)-violating asymmetry is given by

\[
A_{T}^{CP} \propto \sin \phi \cos \delta. \tag{30.31}
\]

The key point here is that one can produce a direct \(CP\) asymmetry only if there is a non-zero strong-phase difference between the two decay amplitudes. However, TP asymmetries are maximal when the strong phase difference is zero. Thus, it may be more promising to look for TP asymmetries than direct \(CP\) asymmetries in \(D\) and \(B\) decays.

Consider the weak decay of a \(D\) meson into a pair of vector mesons, \(D \rightarrow V_{1}(k_{1}, \epsilon_{1})V_{2}(k_{2}, \epsilon_{2})\), where \(k_{1}\) and \(\epsilon\) (\(k_{2}\) and \(\epsilon_{2}\)) denote the polarization and momentum of \(V_{1}\) (\(V_{2}\)). For example, in the decay of \(D^{+} \rightarrow K^{*0}(k_{1}, \epsilon_{1})K^{*+}(k_{2}, \epsilon_{2})\), one can then study the triple correlation

\[
A_{T} = \frac{N(k_{1} \cdot (\epsilon_{1} \times \epsilon_{2}) > 0) - N(k_{1} \cdot (\epsilon_{1} \times \epsilon_{2}) < 0)}{N(k_{1} \cdot (\epsilon_{1} \times \epsilon_{2}) > 0) + N(k_{1} \cdot (\epsilon_{1} \times \epsilon_{2}) < 0)} \tag{30.32}
\]

and the true \(T\) asymmetry, \(A_{T}^{CP} \equiv \frac{1}{2}(A_{T} + \overline{A_{T}})\), can be studied by considering the \(CP\) conjugate decay of \(D^{-} \rightarrow K^{*0}K^{*-}\). \(CP\) symmetry is violated if \(A_{T}^{CP} \neq 0\).

<table>
<thead>
<tr>
<th>CP Violation</th>
<th>Reaction</th>
<th>Event</th>
<th>Comment</th>
</tr>
</thead>
<tbody>
<tr>
<td>(D^0D^0 \rightarrow [\gamma_4(\text{semileptonic})(CP \text{ eigenstates})])</td>
<td>26280</td>
<td>Measure mixing-dependent (CP) violation, (CP) asymmetry determined to 1%</td>
<td></td>
</tr>
<tr>
<td>(\psi(3770) \rightarrow (\text{semileptonic})(CP \text{ eigenstates}))</td>
<td>136000</td>
<td>Measure magnitude of (CP) violating amplitude to 0.5 %</td>
<td></td>
</tr>
<tr>
<td>(\psi(3770) \rightarrow (CP \pm \text{ eigenstates})(CP \pm \text{ eigenstates}))</td>
<td>16000</td>
<td>Sensitive to phase of direct (CP) violating amplitude (1.0%)</td>
<td></td>
</tr>
</tbody>
</table>

Table 30.3: Summary of \(CP\) violation measurements at BES-III