Nonleptonic Charm Decays

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1 Present Status and Implication for QCD

The SM's electroweak phenomenology of charm changing transitions appears dull with the CKM parameters well-known due to three-family unitarity constraints, $D^0 - \bar{D}^0$ oscillations being slow, **CP** asymmetries small at best and loop driven decays extremely rare with huge backgrounds due to long distance dynamics. Yet this very dullness can be utilized to gain new insights into nonperturbative dynamics, make progress in establishing theoretical control over them and calibrate our theoretical tools for B studies.

The issue at stake here is *not* whether QCD is the theory of the strong forces – there is no alternative – but our ability to perform calculations. Charm hadrons can act here as a bridge between the worlds of light flavours – as carried by u, d and s quarks with masses lighter or at most comparable to Λ_{QCD} and described by chiral perturbation theory – and that of the bona fide heavy b quark with $\Lambda_{QCD} \ll m_b$ treatable by heavy quark theory. Only lattice QCD (LQCD) carries the promise for a truly quantitative treatment of charm hadrons that can be improved *systematically*. Furthermore LQCD is the only framework available that allows to approach charm from lower as well as higher mass scales, which involves different aspects of nonperturbative dynamics and thus – if successful – would provide impressive validation.

At present such a program can be carried most explicitly for exclusive semileptonic decays of charm hadrons as described in detail in Sect.26.2, especially since lattice LQCD is reaching a stage where it can make rather accurate predictions for such modes. The theoretical challenges posed by nonleptonic decays are obviously more formidable. The complexities increase considerably for exclusive nonleptonic transitions, in particular due to the importance of final state interactions (FSI), which are much harder to bring under theoretical control even by using start-of-the-art LQCD.

Yet there are some strong motivations for obtaining a reliable description of exclusive nonleptonic charm decays:

- Their dynamics is largely determined by the transition region from the perturbative to the nonperturbative domain. Thus we can gain novel insights there. One should also not give up hope for a future theoretical breakthrough in LQCD (or the

[1]Sections 1 and 2 have been revised by I.Bigi with many additions

advent of another similarly powerful theoretical technology) allowing us to extract numerically reliable lessons.

- The most sensitive probes for New Physics are **CP** asymmetries in nonleptonic channels. The search strategies and subsequent interpretations depend on hadronic matrix elements, FSI and their phases. As already indicated we do not know how to compute them, yet one can profit here from a pragmatic exercise in 'theoretical engineering': providing a phenomenological, yet comprehensive framework for a host of charm modes allows to extract quantitative information on hadroniic matrix elements and FSI phases and evaluate their reliability through overconstraints. The huge datasets already obtained by the B factories, CLEO-c and BESII and to be further expanded including also future BESIII studies will be of essential help here.

- Analogous decays of B are being studied also as a mean to extract the complex phase of V_{ub}. One could hope that D decays might serve as a validation analysis.

- Carefully analysing branching ratios can teach us novel lessons on light-flavour hadron spectroscopy, like on characteristics of some resonances like the scalar mesons or on the η and η' mixing and a possible non-$\bar{q}q$ component in them.

2 Theoretical Review

2.1 The Effective Weak Hamiltonian

The theoretical description starts from constructing an effective $\Delta C \neq 0$ Hamiltonian through an operator product expansion (OPE) in terms of *local* operators O_i and their coefficients c_i:

$$\langle f|\mathcal{H}_{eff}|D\rangle = \frac{G_F}{\sqrt{2}} V_{CKM} \sum_i c_i(\mu)\langle f|O_i|D\rangle(\mu) \tag{1}$$

The *auxiliary* scale μ has been introduced – and this is a central element of the Wilsonian prescription for the OPE – to separate contributions from long and short distance dynamics: long distance $> 1/\mu >$ short distance. Degrees of freedom with typical mass scales above μ are integrated out into the coefficients c_i typically using perturbation theory, while degrees of freedom with scales below μ remain dynamical and are contained in the operators O_i. Nonperturbative dynamics enters through their hadronic expectation values.

Observables of course cannot depend on the choice μ at all. I.e., the μ dependence of the coefficients has to cancel against that of the matrix elements, when one does a complete calculation. Yet in practice one has to keep the following in mind:

- The perturbatively treated coefficients contain also the strong coupling α_S. To keep it in the perturbative domain, one needs

$$\mu \gg \Lambda_{QCD} \tag{2}$$

- Yet at the same time one does not want to choose too high a value for μ, since it also provides the momentum cut-off in the hadronic wave function with which the matrix element is evaluated.

These two contravening requirements can be met by $\mu \sim 1 - 1.5$GeV, which happens to be close to the charm quark mass. Thus $\mu = m_c$ provides a reasonable ansatz. In practice one has to rely on additional approximations of various kinds, which causes the computed rates to contain some sensitivity at least to μ, which can provide one gauge for the reliability of the result.

We do not know yet how to calculate these hadronic matrix elements from QCD's first principles in a numerically accurate way, although several different 'second generation' theoretical technologies have been brought to bear on them: $1/N_C$ expansions, QCD sum rules and lattice QCD. While there is reasonable hope that the latter will be validated in (semi)leptonic D decays, exclusive nonleptonic transitions provide qualitatively new challenges.

While in the SM the weak decays are driven by charged currents, the intervention of QCD affects the strength of the charged current product and induce a product of effective neutral currents in a way that depends on μ. For Cabibbo allowed transitions, one can write down the effective weak Lagragian

$$\mathcal{L}_{eff}^{\Delta C=1}(\mu = m_c) = -\frac{G_F}{\sqrt{2}} V_{ud} V_{cs}^* \cdot [c_- O_- + c_+ O_+] \ , \tag{3}$$

$$O_\pm = \frac{1}{2} \left[(\bar{s}_L \gamma_\nu c_L)(\bar{u}_L \gamma_\nu d_L) \right] \pm (\bar{u}_L \gamma_\nu c_L)(\bar{s}_L \gamma_\nu d_L)] \ , \tag{4}$$

which is conveniently rewritten as follows:

$$\mathcal{L}_{eff}^{\Delta C=1}(\mu = m_c) = -\frac{G_F}{\sqrt{2}} V_{ud} V_{cs}^* \cdot [c_1 O_1 + c_2 O_2] \ ,$$

$$O_1 = (\bar{s}_L \gamma_\nu c_L)(\bar{u}_L \gamma_\nu d_L) \ , \quad O_2 = (\bar{u}_L \gamma_\nu c_L)(\bar{s}_L \gamma_\nu d_L) \ , \tag{5}$$

with

$$c_1 = \frac{1}{2}(c_+ + c_-) \ , \quad c_2 = \frac{1}{2}(c_+ - c_-) \ . \tag{6}$$

Using different schemes one typically gets [1]:

$$c_1(m_c) = 1.25 \pm 0.03 \ , \quad c_2(m_c) = -0.48 \pm 0.05 \tag{7}$$

2.2 Factorization and First Generation Theoretical Technologies

All decay amplitudes can then be expressed as linear combinations of two terms:

$$\mathcal{A}(D \to f) \propto a_1 \langle f | J_\mu^{(ch)} J'^{(ch)\mu} | D \rangle + a_2 \langle f | J_\mu^{(neut)} J'^{(neut)\mu} | D \rangle \ , \tag{8}$$

with

$$a_1 = c_1 + \xi c_2 \ , \quad a_2 = c_2 + \xi c_1 \ . \tag{9}$$

It should be noted that the quantities c_1 and c_2 on one hand and ξ on the other are of *completely different* origin despite their common appearance in a_1 and a_2: while $c_{1,2}$ are determined by short-distance dynamics, ξ parametrizes the impact of long distance dynamics on the size of matrix elements including effects due to final state interactions (FSI). Eq.(9) contains two very important implicit assumptions, namely that the value of ξ is the same in the expression for a_1 and a_2 and that it does not depend on the final state.

A very convenient ansatz is to write the nonleptonic transition matrix element as a product of two simpler matrix elements [2]

$$\langle f|J_\mu J'^\mu|D\rangle \equiv \langle f_1 f_2|J_\mu J'^\mu|D\rangle \simeq \langle f_1|J_\mu|0\rangle\langle f_2|J'^\mu|D\rangle \ , \tag{10}$$

where f_1 and f_2 are "*effective particles*" that can contain any number of final state particles. The basic assumption here is that the colour flow mediated by gluon exchanges *between* the two 'clusters' $0 \to f_1$ and $D \to f_2$ can be ignored and all the strong interaction effects lumped into two simpler transition amplitudes. Clearly this factorization ansatz can be only an approximation rather than an identity. One should also note that Eq.(11) is μ dependent; i.e. changing the value of μ will transform factorized contributions into non-factorized ones and vice versa. The best chance for this ansatz to represent a decent approximation is for the separation scale μ to be around ordinary hadronic scales of about 1 GeV. This values happens to be close to m_c, yet that is a coincidence, since heavy quark masses are extraneous to QCD.

Besides these two types of diagrams which are usually referred as color favored and color suppressed diagrams, other types of considerations are the weak annihilation (WA) contributions including annihilation and exchange diagrams where the matrix element is approximately written as

$$\langle f_1 f_2|J_\mu J'^\mu|D\rangle \simeq \langle f_1 f_2|J_\mu|0\rangle\langle 0|J'^\mu|D\rangle \ . \tag{11}$$

Having assumed factorization we have greatly restricted the number of free parameters. The amplitudes $\langle f_1|J_{i\mu}|0\rangle$ and $\langle f_2|J_i'^\mu|D\rangle$ can be taken from data on (semi)leptonic D decays at least in principle though in practice that information is augmented by some theoretical arguments. The two quantities $a_{1,2}$ are then treated as free parameters fitted from experiment, although in practice again some theoretical judgment has to be applied concerning if and to which degree WA diagrams are included in addition to the spectator diagrams and corrections for FSI have to be applied.

Such an analysis was first carried out by Bauer, Stech and Wirbel from charmed meson two-body decays, yielding [3]

$$a_1|_{exp} \simeq 1.2 \pm 0.1 \ , \quad a_2|_{exp} \simeq -0.5 \pm 0.1 \tag{12}$$

to be compared with the theoretical expectations

$$a_1|_{QCD} \simeq 1.25 - 0.48\xi \ , \quad a_2|_{QCD} \simeq -0.48 + 1.25\xi \ . \tag{13}$$

It is remarkable that with just two fit parameters one can get a decent description of a host of nonleptonic rates. However one might say that those parameters have the wrong values: naively just counting colours one expects $\xi \simeq 1/N_C = 1/3$ and thus $a_1|_{QCD} \simeq 1.09$ and $a_2|_{QCD} \simeq -0.06$; for a_2 this is inconsistent with the experimental fit value. $\xi \simeq 0$ would reconcile Eqs.12 and 13.

2.3 The $1/N_C$ ansatz

The fit result $\xi \simeq 0$ leads to an intriguing speculation that these weak two-body decays can be described more rigorously through $1/N_C$ expansions [4]. They are invoked to calculate hadronic matrix elements. The procedure is the following: One employs the effective weak transition operator $\mathcal{L}_{eff}(\Delta C = 1)$ given explicitly in Eq.(3); since it describes short distance dynamics, one has kept $N_C = 3$ there. Then one expands the matrix element for a certain transition driven by this operators in $1/N_C$

$$\mathcal{A}(D \to f) = \langle f|\mathcal{L}_{eff}(\Delta C = 1)|D\rangle = \sqrt{N_C}\left(b_0 + \frac{b_1}{N_C} + \mathcal{O}(1/N_C^2)\right) \, . \tag{14}$$

Using the rules for $1/N_C$ expansions, it is easy to show that the following simplifying properties hold for the contributions leading in $1/N_C$:

- one has to consider *valence* quark wave functions only;
- *factorization* holds;
- *WA* has to be ignored as have *FSI*.

To leading order in $1/N_C$ only the term b_0 is retained; then one has effectively $\xi = 0$ since $\xi \simeq 1/N_C$ represents a higher order contribution. However the next-to-leading term b_1 is in general beyond theoretical control. $1/N_C$ expansions therefore do not enable us to decrease the uncertainties *systematically*.

The $N_C \to \infty$ prescription is certainly a very compact one with transparent rules, and it provides not a bad first approximation – but not more. One can ignore neither FSI nor WA completely.

2.4 Treatment with QCD sum rules

A treatment of $D \to PP$ and $D \to PV$ decays based on a judicious application of QCD sum rules was developed in a series of papers [5]. The authors analyzed four-point correlation functions between the weak Lagrangian $\mathcal{L}(\Delta C = 1)$ and three currents. As usual an OPE is applied to the correlation function in the Euclidean region; nonperturbative dynamics is incorporated through condensates $\langle 0|m\bar{q}q|0\rangle$, $\langle 0|G \cdot G|0\rangle$ etc., the numerical values of which are extracted from other *light*-quark systems. They extrapolated their results to the Minkowskian domain through a (double) dispersion relation and succeed in finding a stability range for matching it with phenomenological hadronic expressions.

The analysis has some nice features:
$\oplus$ It has a clear basis in QCD, and includes, in principle at least, nonperturbative dynamics in a well-defined way.
$\oplus$ It incorporates different quark-level processes – external and internal W emission, WA and Pauli interference – in a natural manner.
$\oplus$ It allows to include nonfactorizable contributions systematically.

In practice, however, it suffers from some shortcomings at the same time:
$\ominus$ The charm scale is not sufficiently high that one could have full confidence in the various extrapolations undertaken.
$\ominus$ To make these very lengthy calculations at all manageable, some simplifying assumptions had to be made, like $m_u = m_d = m_s = 0$ and $SU(3)_{Fl}$ breaking beyond $m_K > m_\pi$ had to be ignored; in particular $\langle 0|\bar{s}s|0\rangle = \langle 0|\bar{d}d|0\rangle = \langle 0|\bar{u}u|0\rangle$ was used. Thus one cannot expect $SU(3)_{Fl}$ breaking to be reproduced correctly.
$\ominus$ *Prominent* FSI that vary rapidly with the energy scale – like effects due to narrow resonances - cannot be described in this treatment; for an extrapolation from the Euclidean to the Minkowskian domain amounts to some averaging or 'smearing' over energies.

A statement that the predictions did not provide an excellent fit to the data on about twenty-odd D^0, D^+ and D_s^+ modes – while correct on the surface, especially when $SU(3)_{Fl}$ breaking is involved – misses the main point:

- No a priori model assumption like factorization had to be made.
- The theoretical description does not contain any free parameters in principle, though in practice there is leeway in the size of some decay constants.

2.5 Modern Developments

As the data improved, the BSW prescription became inadequate, however most subsequent attempts to describe nonleptonic decays in the D system – except for the sum rules approach sketched above – use the assumption of naive factorization as a starting point.

Improvements and generalizations of the BSW description have been made in three areas:

1. Different parameterizations for the q^2 dependence of the form factors are used and different evaluations of their normalization are made. This is similar to what was addressed in our discussion of exclusive semileptonic decays. One appealing suggestion has been suggested to use only those expressions for form factors that asymptotically – i.e. for m_c, $m_s \to \infty$ – exhibit heavy quark symmetry.
2. Attempts have been made to incorporate FSI more reliably. Non-factorized contributions in general have been considered.
3. Contributions due to WA and Penguin operators have been included.

Two frameworks that are more firmly based on QCD than quark models have been developed to treat two-body decays of B mesons, namely 'QCD factorization' [6] and 'pQCD' [7]. While there is little reason to expect the more aggressive pQCD approach to work already for charm decays, a treatment based on QCD factorization is worth a try despite the charm mass barely exceeding ordinary hadronic scales. To illustrate the present status the branching ratios of $D \to \pi\pi$ decays as inferred from naive factorization and QCD factorization approaches are listed here [8]:

$$BR(D^0 \to \pi^+\pi^-) = \begin{cases} 1.86 \times 10^{-3} \ , & (Naive \ \ Factorization) \\ 1.69 \times 10^{-3} \ , & (QCD \ \ Factorization) \\ (1.364 \pm 0.032) \times 10^{-3} \ , & (PDG06[9]) \end{cases}$$

$$BR(D^+ \to \pi^+\pi^0) = \begin{cases} 1.68 \times 10^{-3} \ , & (Naive \ \ Factorization) \\ 1.94 \times 10^{-3} \ , & (QCD \ \ Factorization) \\ (1.28 \pm 0.09) \times 10^{-3} \ , & (PDG06[9]) \end{cases}$$

$$BR(D^0 \to \pi^0\pi^0) = \begin{cases} 2.44 \times 10^{-5} \ , & (Naive \ \ Factorization) \\ 2.06 \times 10^{-5} \ , & (QCD \ \ Factorization) \\ (7.9 \pm 0.8) \times 10^{-4} \ . & (PDG06[9]) \end{cases}$$

Both the naive factorization and the QCD factorization predictions on $D^0 \to \pi^+\pi^-$ and $D^+ \to \pi^+\pi^0$ can accommodate the experimental results to the same order, while the prediction on $D^0 \to \pi^0\pi^0$ is about forty times smaller than the experimental result. For proper perspective one should note that the modes $D^0 \to \pi^+\pi^-$ and $D^+ \to \pi^+\pi^0$ are described by color *favored* tree diagram T, whereas $D^0 \to \pi^0\pi^0$ is dominated by a color *suppressed* tree diagram C. Non-factorizable corrections are found to be larger for the latter and at present are beyond theoretical control.

In summary, a theoretical description of exclusive nonleptonic decays of charmed mesons based on general principles is not yet possible. Even though the short distance contributions can be calculated and the effective weak Hamiltonian has been constructed at next-to-leading order, the evaluation of its matrix elements requires nonperturbative techniques. Some decent phenomenological descriptions have been achieved, yet realistically few frameworks can provide some openings for systematic improvements, especially when they are applied to multi-body channels.

2.6 Symmetry Analysis

2.6.1 Isospin SU(2) Symmetry

Symmetry based arguments are a powerful tool in our theoretical arsenal. Isospin invariance should hold on the $\mathcal{O}(1\%)$ level, and no evidence to the contrary has been found. Take two-body decays as an example, it leads to triangle relations among the decay amplitudes:

$$\mathcal{A}(D^0 \to \pi^+\pi^-) + \sqrt{2}\mathcal{A}(D^0 \to \pi^0\pi^0) - \sqrt{2}\mathcal{A}(D^+ \to \pi^+\pi^0) = 0 \ , \qquad (15)$$

$$\mathcal{A}(D^0 \to K^-\pi^+) + \sqrt{2}\mathcal{A}(D^0 \to \bar{K}^0\pi^0) - \mathcal{A}(D^+ \to \bar{K}^0\pi^+) = 0 \ , \qquad (16)$$

$$\mathcal{A}(D^0 \to \pi^+ K^{*-}) + \sqrt{2}\mathcal{A}(D^0 \to \pi^0 \bar{K}^{*0}) - \mathcal{A}(D^+ \to \pi^+ \bar{K}^{*0}) = 0 \ , \tag{17}$$

$$\mathcal{A}(D^0 \to \rho^+ K^-) + \sqrt{2}\mathcal{A}(D^0 \to \rho^0 \bar{K}^0) - \mathcal{A}(D^+ \to \bar{K}^0 \rho^+) = 0 \ . \tag{18}$$

The measured rates tell us that these amplitudes possess large relative phases due to strong FSI. Considering the three $D \to \pi\pi$ modes, the transition amplitudes can be decomposed into

$$\mathcal{A}(D^0 \to \pi^+\pi^-) = \sqrt{\tfrac{2}{3}}\mathcal{A}_0 + \sqrt{\tfrac{1}{3}}\mathcal{A}_2 \ , \tag{19}$$

$$\mathcal{A}(D^0 \to \pi^0\pi^0) = \sqrt{\tfrac{1}{3}}\mathcal{A}_0 - \sqrt{\tfrac{2}{3}}\mathcal{A}_2 \ , \tag{20}$$

$$\mathcal{A}(D^+ \to \pi^+\pi^0) = \sqrt{\tfrac{3}{2}}\mathcal{A}_2 \ . \tag{21}$$

The subscripts 0 and 2 of the $\mathcal{A}$ describe the isospin $I = 0$ and 2 part of the $\pi\pi$ system. From the experimental data, the amplitude ratio $|A_2/A_0|$ and the relative phase $\delta = \delta_2 - \delta_0$ were extracted [10]

$$|A_2/A_0| = 0.72 \pm 0.13 \pm 0.11 \ , \quad \cos\delta = 0.14 \pm 0.13 \pm 0.09 \ . \tag{22}$$

2.6.2 Flavor SU(3) Symmetry and Its Breaking

It would seem tempting to argue that SU(3) flavor symmetry holds to within, say, 20 - 30 %. This, however, does not seem to be the case, at least not for exclusive channels, as can be read off most dramatically from the following comparison

$$\frac{\mathcal{A}(D^0 \to K^+K^-)}{\mathcal{A}(D^0 \to \pi^+\pi^-)} \simeq 1.8 \tag{23}$$

rather than unity. The significant SU(3) symmetry violation may come from the finite strange quark mass, FSI and resonances [11]. There are indications, however, that for inclusive rates SU(3) flavor breaking does not exceed the 20% level [12].

3 Two-Body Decays

Two-body modes in charmed meson nonleptonic decays have been drawn much attention since 1980s because they have the following advantages, as compared with multi-body ones, that put them to a heated place:

- The nonleptonic decays of charm mesons have been observed to proceed mainly through two-body channels, if one counts resonances as one body. A great number of precise experimental data on two-body decays, including branching ratios of about 60 decay modes, have been accumulated in the *Particle Data Group* [9].

- The phase space is trivial and the number of form factors are quite limited.

- There are less colour sources in the form of quarks and antiquarks, less different combinations for colour flux tubes to form.

- Quite a number of two-body modes allow for sizeable momentum transfers thus hopefully reducing the predominance of long-distance dynamics.

- They are one class of nonleptonic decays where one can harbor reasonable hope of some success. It is not utopian to expect lattice QCD to treat these transitions some day in full generality. Such results will however be reliable only, if obtained with incorporating fully dynamical fermions – i.e. without "quenching" and without relying on a $1/m_c$ expansion.

3.1 Kinematics and Topologies of Amplitudes

In the center-of-mass frame, the differential decay rate for n-body charmed meson decay is

$$d\Gamma = \frac{1}{2m_D}(\prod_{i=1}^{n} \frac{d^3p_i}{(2\pi)^3}\frac{1}{2E_i})|\mathcal{A}(m_D \to \{p_1, p_2, \cdots, p_n\})|^2(2\pi)^4\delta^4(p_D - \sum_{i=1}^{n} p_i). \tag{24}$$

If the number of final state particles is set to two, one can easily conduct the integral over the phase space to obtain the decay rate

$$\Gamma(D \to f_1 f_2) = \frac{p}{8\pi M_D^2}|\mathcal{A}|^2 \ , \tag{25}$$

where

$$p = \frac{\sqrt{(M_D^2 - (m_1 + m_2)^2)(M_D^2 - (m_1 - m_2)^2)}}{2M_D}$$

denotes the center-of-mass 3-momentum of each final particle. The branching fraction of $D \to f_1 f_2$ transition is then defined as the ratio of the decay rate to the full width of D meson

$$B(D \to f_1 f_2) = \frac{\Gamma(D \to f_1 f_2)}{\Gamma(D)} \ . \tag{26}$$

The amplitude $\mathcal{A}$ can be decomposed into six distinct quark-graph topologies [13]: (1) color-favored tree amplitude T, (2) color-suppressed tree amplitude C, (3) W-exchange amplitude E, (4) W-annihilation amplitude A, (5) horizontal W-loop amplitude P and (6) vertical W-loop amplitude D. The penguin diagrams P and D play little role in practice because the relation of the CKM matrix elements $V_{cs}^* V_{us} \approx -V_{cd}^* V_{ud}$ will result in cancellations among them.

3.2 $D \to PP$, $D \to PV$ and $D \to VV$ Decays

Among the measured experimental data on charm nonleptonic two-body decays, charmed mesons decaying to two pseudoscalar mesons ($D \to PP$), to one pseudoscalar and one vector meson ($D \to PV$) and to two vector mesons ($D \to VV$) are of dominant quality. The light pseudoscalar and vector mesons are two classes of particles that have been made clear on their basic properties like mass, lifetime, width, quark component and decay rate. The form factors of charmed mesons transiting to light pseudoscalar and vector mesons have been calculated in a variety of theoretical models. Based on the resulting form factors, most predictions on charmed meson semileptonic decays are consistent with experimental data, as shown in Section 26.2. As a result, it is an ideal place to test the factorization assumption and understand the mysteries of FSI and unfactorizable contributions.

Provided that the definitions of form factors are adopted as in Equations (26.36) and (26.42), the four relevant amplitudes for $D \to P_1P_2$ in the formalism of factorization approach read

$$T = i\frac{G_F}{\sqrt{2}}V_{q_1q_2}V^*_{cq_3}a_1f_{P_1}(m^2_D - m^2_2)F_0^{D\to P_2}(m^2_1), \tag{27}$$

$$C = i\frac{G_F}{\sqrt{2}}V_{q_1q_2}V^*_{cq_3}a_2f_{P_1}(m^2_D - m^2_2)F_0^{D\to P_2}(m^2_1), \tag{28}$$

$$E = i\frac{G_F}{\sqrt{2}}V_{q_1q_2}V^*_{cq_3}a_2f_D(m^2_1 - m^2_2)F_0^{P_1P_2}(m^2_D), \tag{29}$$

$$A = i\frac{G_F}{\sqrt{2}}V_{q_1q_2}V^*_{cq_3}a_1f_D(m^2_1 - m^2_2)F_0^{P_1P_2}(m^2_D). \tag{30}$$

The amplitudes for $D \to PV$ are a little more complicated than $D \to PP$, for one should distinguish the pseudoscalar or the vector final state where the spectator quark enters. Using a subscript P or V to denote the spectator quark containing in pseudoscalar or vector final meson, one can read out the amplitudes

$$T_V = 2\frac{G_F}{\sqrt{2}}V_{q_1q_2}V^*_{cq_3}a_1f_Pm_V(\varepsilon^*\cdot p_D)A_0^{D\to V}(m^2_P), \tag{31}$$

$$T_P = 2\frac{G_F}{\sqrt{2}}V_{q_1q_2}V^*_{cq_3}a_1f_Vm_V(\varepsilon^*\cdot p_D)F_+^{D\to P}(m^2_V), \tag{32}$$

$$C_V = 2\frac{G_F}{\sqrt{2}}V_{q_1q_2}V^*_{cq_3}a_2f_Pm_V(\varepsilon^*\cdot p_D)A_0^{D\to V}(m^2_P), \tag{33}$$

$$C_P = 2\frac{G_F}{\sqrt{2}}V_{q_1q_2}V^*_{cq_3}a_2f_Vm_V(\varepsilon^*\cdot p_D)F_+^{D\to P}(m^2_V), \tag{34}$$

$$E = 2\frac{G_F}{\sqrt{2}}V_{q_1q_2}V^*_{cq_3}a_2f_Dm_V(\varepsilon^*\cdot p_D)A_0^{PV}(m^2_D), \tag{35}$$

$$A = 2\frac{G_F}{\sqrt{2}}V_{q_1q_2}V^*_{cq_3}a_1f_Dm_V(\varepsilon^*\cdot p_D)A_0^{PV}(m^2_D). \tag{36}$$

The decay modes of charm to two vector meson final states have a richer structure than the decays with at least one pseudoscalar in the final state.

$$T(D \to V_1V_2) = \frac{G_F}{\sqrt{2}}V_{q_1q_2}V^*_{cq_3}a_1[if_{V_1}m_1(m_D + m_2)A_1^{D\to V_2}(m^2_1)\varepsilon^*_1\cdot\varepsilon^*_2$$

$$
\begin{aligned}
& -i\frac{1}{m_D+m_2}f_{V_1}m_1A_2^{D\to V_2}(m_1^2)\varepsilon_1^*\cdot(p_D+p_2)\varepsilon_2^*\cdot(p_D-p_2) \\
& -\frac{2}{m_D+m_2}f_{V_1}m_1V^{D\to V_2}(m_1^2)\epsilon_{\mu\nu\alpha\beta}\varepsilon_1^{*\mu}\varepsilon_2^{*\nu}p_D^\alpha p_2^\beta] \ , \qquad (37)
\end{aligned}
$$

$$
\begin{aligned}
C(D\to V_1V_2) \;=\; & \frac{G_F}{\sqrt{2}}V_{q_1q_2}V^*_{cq_3}a_2[if_{V_1}m_1(m_D+m_2)A_1^{D\to V_2}(m_1^2)\varepsilon_1^*\cdot\varepsilon_2^* \\
& -i\frac{1}{m_D+m_2}f_{V_1}m_1A_2^{D\to V_2}(m_1^2)\varepsilon_1^*\cdot(p_D+p_2)\varepsilon_2^*\cdot(p_D-p_2) \\
& -\frac{2}{m_D+m_2}f_{V_1}m_1V^{D\to V_2}(m_1^2)\epsilon_{\mu\nu\alpha\beta}\varepsilon_1^{*\mu}\varepsilon_2^{*\nu}p_D^\alpha p_2^\beta] \ . \qquad (38)
\end{aligned}
$$

The terms proportional to A_1, A_2 and V represent S, longitudinal D and P waves respectively. The omitted expressions for exchange and annihilation topologies contain the vector-to-vector form factor.

Due to the historical phenomenological analysis, we can draw some general conclusions on the above factorization formalism, which serve as guides for the studies of the other charmed meson decay modes like two-body final states containing scalar (S), axial-vector (A), tensor (T) and multi-body final states.

- Nonfactorizable corrections which result from spectator interactions, FSI, resonance effects and so on are found to be significant [14]. Some phenomenological models based on one-particle-exchange method [15], resonance formation [16], the combination of heavy quark effective theory and chiral perturbation theory [17] have been developed to make some insights into them. The resonance formation via $q\bar{q}$ resonances is probably the most important one to hadronic charm decays owing to the existence of an abundant spectrum of resonances known to exist at energy close to the mass of charmed mesons. Most of the properties of resonances follow from unitarity so that the effects of resonance-induced nonfactorizable contributions can be described in a model independent manner in terms of the masses and the decay widths of the nearby resonances [18].

- The parameters a_1 and a_2 were found to be not universal but process or class dependent. For illustration purposes, we take some examples

$$
a_1(\mu) = c_1(\mu) + (\frac{1}{N_c} + \chi_1(\mu))c_2(\mu) \ , \qquad (39)
$$

$$
a_2(\mu) = c_2(\mu) + (\frac{1}{N_c} + \chi_2(\mu))c_1(\mu) \ , \qquad (40)
$$

  with $\chi_1(\mu)$ and $\chi_2(\mu)$ partially denoting nonfactorizable effects in the case of $N_c = 3$. $\chi_2(\mu)$ was determined to be [19]

$$
\begin{aligned}
& \chi_2(D\to\bar{K}\pi) \simeq -0.33 \ , \\
& \chi_2(D\to\bar{K}^*\pi) \simeq -(0.45\sim 0.55) \ , \\
& \chi_2(D\to\bar{K}^*\rho) \simeq -(0.6\sim 0.65) \ . \qquad (41)
\end{aligned}
$$

- The light meson to light meson form factors involving in the above formula are believed to be negligibly small. Thus the factorizable formalism of the exchange and annihilation diagrams make no effect on the overall amplitudes. The main contributions of these diagrams may result from the nonfactorizable parts. Through intermediate states, they relate to the tree diagram T and color-suppressed diagram C [18, 20]. As a consequence, they have sizable magnitudes comparable to the T and C amplitudes and large strong phases relative to the T amplitude as illustrated in a SU(3) flavor symmetry analysis [21, 22]. Especially in the case of decay mode $D^0 \to K^0\overline{K}^0$ which transits thoroughly through $E - E$ diagram representation, the factorizable contribution is too trivial to be consistent with the experimental branching ratio $B = (7.1 \pm 1.9) \times 10^{-4}$. Many studies have been perform on this decay [23] and it is found that the nonfactorizable correction of E diagram can account for experimental data to the same order.

A variety of charmed meson decay processes calculated in some literatures are presented in Table 1 for $D \to PP$ decays and in Table 2 for $D \to PV$ decays. It is noted that all of these considerations have introduced more or less free parameters to describe the nonfactorizable contributions and need to be fitted out from experiments. For $D \to VV$ decays, some research articles in ten years ago can be found in [17, 26, 27, 28].

3.3 $D \to SP$ Decays

Scalar meson production measurements in charm decays are now available from the dedicated experiments conducted at ARGUS [29], CLEO [30], E687 [31], E691 [32], E791 [33], FOCUS [34], and BaBar [35]. Specifically, the decays $D \to f_0\pi(K)$, $D \to a_0\pi(K)$, $D \to \overline{K}_0^*\pi$ and $D^+ \to \sigma\pi^+$ have been observed through Dalitz plot analysis of three-body decays. The results of various experiments are summarized in Table 3 where the products of $\mathcal{B}(D \to SP_3)$ and $\mathcal{B}(S \to P_1P_2)$ are listed. In order to extract the branching ratios for $D \to f_0P$, one shall use the the result from a recent analysis [36]: $\Gamma(f_0 \to \pi\pi) = (64 \pm 8)MeV$, $\Gamma(f_0 \to K\overline{K}) = (12 \pm 1)MeV$ and $\Gamma_{f_0\ total} = (80 \pm 10)MeV$. Therefore, one has

$$\mathcal{B}(f_0(980) \to K^+K^-) = 0.08 \pm 0.01\,, \qquad \mathcal{B}(f_0(980) \to \pi^+\pi^-) = 0.53 \pm 0.09\,. \quad (42)$$

For $D \to a_0P$, one can apply the PDG average $\Gamma(a_0 \to K\overline{K})/\Gamma(a_0 \to \pi\eta) = 0.183 \pm 0.024$ [9] to obtain

$$\begin{aligned} \mathcal{B}(a_0^0(980) \to \pi^0\eta) &= 0.845 \pm 0.017 \\ \mathcal{B}(a_0^+(980) \to K^+\overline{K}^0) &= \mathcal{B}(a_0^-(980) \to K^-K^0) = 0.155 \pm 0.017\,, \\ \mathcal{B}(a_0^0(980) \to K^+K^-) &= 0.078 \pm 0.009\,. \end{aligned} \quad (43)$$

For $D \to K_0^*(1430)P$, $\mathcal{B}(K_0^*(1430) \to K^+\pi^-) = \frac{2}{3}(0.93 \pm 0.10)$ can be put to application [9]. It is worth noting that some of the scalar meson decays are not shown in PDG.

Table 1: *Predicted branching ratios for $D \to PP$ (in 10^{-2}). Most decay modes involving a neutral K meson are given as K_S^0 in PDG06 instead of $\overline{K}^0$ in PDG04 which are presented here as well.*

Decay Modes	Buccella *et al.* [24]	Du *et al.*[15]	Wu *et al.* [25]	PDG 06/04
$D^0 \to K^-\pi^+$	3.847	3.72	3.79 ; 3.80	3.80 ± 0.07
$\to \overline{K}^0\pi^0$	1.310	2.09	2.27 ; 2.24	2.28 ± 0.22 ($pdg04$) 1.14 ± 0.12
$\to \overline{K}^0\eta$			0.80 ; 0.81	0.76 ± 0.11 ($pdg04$) 0.38 ± 0.06
$\to \overline{K}^0\eta'$			1.85 ; 1.88	1.87 ± 0.28 ($pdg04$) 0.91 ± 0.14
$\to \pi^+\pi^-$	0.151	0.149	0.144 ; 0.144	0.1364 ± 0.0032
$\to \pi^0\pi^0$	0.115	0.106	0.078 ; 0.097	0.079 ± 0.008
$\to K^+K^-$	0.424	0.40	0.413 ; 0.413	0.384 ± 0.010
$\to K^0\overline{K}^0$	0.130	0.0573	0.069 ; 0.062	0.071 ± 0.019 ($pdg04$) 0.037 ± 0.007
$\to K^+\pi^-$	0.033	0.0141	0.0150 ; 0.0151	0.0143 ± 0.0004
$\to \eta\pi^0$			0.069 ; 0.068	0.056 ± 0.014
$\to \eta'\pi^0$			0.088 ; 0.091	—
$\to \eta\eta$			0.011 ; 0.016	—
$\to \eta\eta'$			0.026 ; 0.030	—
$\to K^0\pi^0$	0.008	0.0284	0.002 ; 0.005	—
$\to K^0\eta$			0.001 ; 0.002	—
$\to K^0\eta'$			0.0 ; 0.0	—
$D^+ \to \overline{K}^0\pi^+$	2.939		2.76 ; 2.76	2.77 ± 0.18 ($pdg04$) 1.47 ± 0.06
$\to \pi^+\pi^0$	0.185	0.18	0.25 ; 0.19	0.128 ± 0.009
$\to \eta\pi^+$			0.34 ; 0.37	0.35 ± 0.032
$\to \eta'\pi^+$			0.45 ; 0.42	0.53 ± 0.11
$\to K^+\overline{K}^0$	0.764	0.64	0.62 ; 0.62	0.58 ± 0.06 ($pdg04$) 0.296 ± 0.019
$\to K^0\pi^+$	0.053	0.0756	0.012 ; 0.026	—
$\to K^+\pi^0$	0.055	0.0296	0.021 ; 0.023	< 0.042
$\to K^+\eta$			0.011 ; 0.012	—
$\to K^+\eta'$			0.005 ; 0.006	—
$D_s^+ \to \overline{K}^0K^+$	4.623		3.06 ; 3.13	4.4 ± 0.9
$\to \pi^+\eta$	1.131		1.05 ; 1.09	2.11 ± 0.35
$\to \pi^+\eta'$			4.19 ; 4.43	4.7 ± 0.7
$\to \pi^+K^0$	0.373		0.24 ; 0.26	< 0.9
$\to \pi^0K^+$	0.146		0.047 ; 0.090	—
$\to \eta K^+$	0.300		0.055 ; 0.040	—
$\to \eta'K^+$			0.090 ; 0.102	—
$\to K^+K^0$	0.012		0.014 ; 0.010	—

Table 2: *Predicted branching ratios for $D \to PV$ (in 10^{-2}). Most decay modes involving a neutral K meson are given as K_S^0 in PDG06 instead of $\overline{K}^0$ in PDG04 which are presented here as well.*

Decay Modes	Buccella *et al.* [24]	Du *et al.*[15]	Wu *et al.* [25]	PDG 06/04
$D^0 \to K^{*-}\pi^+$	4.656	5.22	5.93 ; 5.97	5.9 ± 0.4 (*pdg*04)
$\to K^-\rho^+$	11.201	11.1	9.99 ; 9.90	10.1 ± 0.8 (*pdg*04)
$\to \overline{K}^{*0}\pi^0$	3.208	2.72	2.72 ; 2.81	2.8 ± 0.4 (*pdg*04)
$\to \overline{K}^0\rho^0$	0.759	1.25	1.49 ; 1.25	$1.55^{+0.12}_{-0.16}$ (*pdg*04)
$\to \overline{K}^{*0}\eta$			1.50 ; 1.94	1.8 ± 0.4 (*pdg*04)
$\to \overline{K}^0\omega$	1.855		2.11 ; 1.80	2.3 ± 0.4 (*pdg*04)
				1.1 ± 0.2
$\to \overline{K}^0\phi$			0.95 ; 0.90	0.94 ± 0.11 (*pdg*04)
$\to K^+K^{*-}$	0.290		0.25 ; 0.25	0.20 ± 0.11
$\to K^-K^{*+}$	0.431		0.43 ; 0.43	0.37 ± 0.08
$\to K^0\overline{K}^{*0}$	0.052		0.08 ; 0.16	< 0.17 (*pdg*04)
				< 0.08
$\to \overline{K}^0K^{*0}$	0.062		0.08 ; 0.16	< 0.09 (*pdg*04)
				< 0.04
$\to \pi^0\phi$	0.105		0.12 ; 0.12	0.074 ± 0.005
$\to \overline{K}^{*0}\eta'$			0.004 ; 0.003	< 0.10 (*pdg*04)
$\to \eta\phi$			0.035 ; 0.034	0.014 ± 0.004
$\to \pi^+\rho^-$	0.485	0.36	0.34 ; 0.35	0.45 ± 0.04
$\to \pi^-\rho^+$	0.706	0.73	0.62 ; 0.61	1.0 ± 0.06
$\to \pi^0\rho^0$	0.216	0.11	0.19 ; 0.16	0.32 ± 0.04
$\to \pi^0\omega$	0.013		0.020 ; 0.003	< 0.026
$\to \eta\omega$			0.13 ; 0.10	
$\to \eta'\omega$			0.0007 ; 0.0003	
$\to \eta\rho^0$			0.0039 ; 0.0015	
$\to \eta'\rho^0$	0.039		0.012 ; 0.009	
$\to K^{*+}\pi^-$	0.025		0.029 ; 0.029	
$\to K^+\rho^-$	0.004		0.016 ; 0.016	
$\to K^{*0}\pi^0$	0.008		0.0052 ; 0.0064	
$\to K^0\rho^0$			0.0069 ; 0.0059	
$\to K^{*0}\eta$			0.0030 ; 0.0041	
$\to K^{*0}\eta'$			0.0 ; 0.0	
$\to K^0\omega$	0.002		0.0076 ; 0.0056	
$\to K^0\phi$			0.0 ; 0.0006	

Table 2: (*continued*) *Predicted branching ratios for* $D \to PV$ *(in* 10^{-2}*). Most decay modes involving a neutral* K *meson are given as* K_S^0 *in PDG06 instead of* $\overline{K}^0$ *in PDG04 which are presented here as well.*

Decay Modes	Buccella *et al.* [24]	Du *et al.*[15]	Wu *et al.* [25]	PDG 06/04
$D^+ \to \overline{K}^{*0}\pi^+$	1.996	1.93	1.96 ; 1.96	1.95 ± 0.19 (*pdg*04)
$\to \pi^+\phi$	0.619		0.64 ; 0.62	0.65 ± 0.07
$\to \overline{K}^0\rho^+$	12.198	7.01	7.56 ; 8.43	6.6 ± 2.5 (*pdg*04)
$\to \pi^+\rho^0$	0.104	0.13	0.088 ; 0.088	0.107 ± 0.011
$\to K^+\overline{K}^{*0}$	0.436		0.44 ; 0.44	0.43 ± 0.06 (*pdg*04)
$\to \overline{K}^0 K^{*+}$	1.515		1.43 ; 1.25	3.1 ± 1.4 (*pdg*04)
				1.6 ± 0.7
$\to K^+\rho^0$	0.029		0.030 ; 0.025	0.025 ± 0.007
$\to K^{*0}\pi^+$	0.027		0.024 ; 0.022	0.030 ± 0.006
$\to K^+\phi$			0.0066 ; 0.0067	< 0.013 (*pdg*04)
$\to \pi^+\omega$			0.57 ; 0.58	< 0.034
$\to \eta\rho^+$			0.24 ; 0.43	< 0.7
$\to \eta'\rho^+$			0.15 ; 0.15	< 0.6
$\to \pi^0\rho^+$	0.451	0.31	0.28 ; 0.35	
$\to K^0\rho^+$	0.042		0.025 ; 0.022	
$\to \pi^0 K^{*+}$	0.057		0.037 ; 0.036	
$\to K^+\omega$			0.012 ; 0.011	
$\to K^{*+}\eta$			0.015 ; 0.015	
$\to K^{*+}\eta'$			0.00014 ; 0.00016	
$D_s^+ \to \overline{K}^{*0}K^+$	4.812		3.34 ; 3.42	3.3 ± 0.9 (*pdg*04)
$\to \overline{K}^0 K^{*+}$	2.467		4.98 ; 4.66	5.3 ± 1.3
$\to \pi^+\rho^0$			0.06 ; 0.06	< 0.07 (*pdg*04)
$\to \pi^+\phi$	4.552		3.08 ; 2.93	4.4 ± 0.6
$\to \pi^+ K^{*0}$	0.445		0.33 ; 0.35	0.65 ± 0.28 (*pdg*04)
$\to K^+\rho^0$	0.198		0.12 ; 0.12	0.26 ± 0.07
$\to K^+\phi$	0.008		0.032 ; 0.033	< 0.06
$\to K^+\omega$	0.178		0.40 ; 0.39	
$\to K^0\rho^+$	1.288		0.91 ; 0.77	
$\to \pi^0 K^{*+}$	0.076		0.13 ; 0.13	
$\to \eta K^{*+}$	0.146		0.038 ; 0.047	
$\to \eta' K^{*+}$			0.068 ; 0.059	
$\to K^{*0}K^+$	0.006		0.0015 ; 0.0015	
$\to K^{*+}K^0$	0.018		0.0076 ; 0.0085	

Table 3: *Experimental branching ratios of various $D \to SP$ decays measured by ARGUS, E687, E691, E791, CLEO, FOCUS and BaBar. For simplicity and convenience, The mass identification for $f_0(980)$, $a_0(980)$ and $K_0^*(1430)$ have been dropped.*

Collaboration	$\mathcal{B}(D \to SP) \times \mathcal{B}(S \to P_1 P_2)$	$\mathcal{B}(D \to SP)$
PDG06	$\mathcal{B}(D^+ \to f_0\pi^+)\mathcal{B}(f_0 \to \pi^+\pi^-) = (2.1 \pm 0.5) \times 10^{-4}$	$\mathcal{B}(D^+ \to f_0\pi^+) = (4.0 \pm 1.2) \times 10^{-4}$
E791	$\mathcal{B}(D^+ \to f_0\pi^+)\mathcal{B}(f_0 \to \pi^+\pi^-) = (1.9 \pm 0.5) \times 10^{-4}$	$\mathcal{B}(D^+ \to f_0\pi^+) = (3.6 \pm 1.1) \times 10^{-4}$
FOCUS	$\mathcal{B}(D^+ \to f_0K^+)\mathcal{B}(f_0 \to K^+K^-) = (3.84 \pm 0.92) \times 10^{-5}$	$\mathcal{B}(D^+ \to f_0K^+) = (4.8 \pm 1.3) \times 10^{-4}$
PDG06	$\mathcal{B}(D^+ \to f_0K^+)\mathcal{B}(f_0 \to \pi^+\pi^-) = (5.7 \pm 3.5) \times 10^{-5}$	$\mathcal{B}(D^+ \to f_0K^+) = (1.1 \pm 0.7) \times 10^{-4}$
FOCUS	$\mathcal{B}(D^+ \to f_0K^+)\mathcal{B}(f_0 \to \pi^+\pi^-) = (6.12 \pm 3.65) \times 10^{-5}$	$\mathcal{B}(D^+ \to f_0K^+) = (1.2 \pm 0.7) \times 10^{-4}$
FOCUS	$\mathcal{B}(D^+ \to a_0^0\pi^+)\mathcal{B}(a_0^0 \to K^+K^-) = (2.38 \pm 0.47) \times 10^{-3}$	$\mathcal{B}(D^+ \to a_0^0\pi^+) = (3.1 \pm 0.7)\%$
E791	$\mathcal{B}(D^+ \to \sigma\pi^+)\mathcal{B}(\sigma \to \pi^+\pi^-) = (1.4 \pm 0.3) \times 10^{-3}$	$\mathcal{B}(D^+ \to \sigma\pi^+) = (2.1 \pm 0.5) \times 10^{-3}$
E791	$\mathcal{B}(D^+ \to \kappa\pi^+)\mathcal{B}(\kappa \to K^-\pi^+) = (4.4 \pm 1.2)\%$	$\mathcal{B}(D^+ \to \kappa\pi^+) = (6.5 \pm 1.9)\%$
E691,E687	$\mathcal{B}(D^+ \to \overline{K}_0^{*0}\pi^+)\mathcal{B}(\overline{K}_0^{*0} \to K^-\pi^+) = (2.3 \pm 0.3)\%$	$\mathcal{B}(D^+ \to \overline{K}_0^{*0}\pi^+) = (3.7 \pm 0.6)\%$
PDG06	$\mathcal{B}(D^+ \to \overline{K}_0^{*0}\pi^+)\mathcal{B}(\overline{K}_0^{*0} \to K^-\pi^+) = (2.41 \pm 0.24)\%$	$\mathcal{B}(D^+ \to \overline{K}_0^{*0}\pi^+) = (3.9 \pm 0.6)\%$
E791	$\mathcal{B}(D^+ \to \overline{K}_0^{*0}\pi^+)\mathcal{B}(\overline{K}_0^{*0} \to K^-\pi^+) = (1.14 \pm 0.16)\%$	$\mathcal{B}(D^+ \to \overline{K}_0^{*0}\pi^+) = (1.8 \pm 0.3)\%$
PDG06	$\mathcal{B}(D^+ \to \overline{K}_0^{*0}K^+)\mathcal{B}(\overline{K}_0^{*0} \to K^-\pi^+) = (3.7 \pm 0.4) \times 10^{-3}$	$\mathcal{B}(D^+ \to \overline{K}_0^{*0}K^+) = (6.0 \pm 0.9) \times 10^{-3}$
PDG06	$\mathcal{B}(D^+ \to f_0(1370)\pi^+)\mathcal{B}(f_0(1370) \to \pi^+\pi^-) = (8 \pm 6) \times 10^{-5}$	
FOCUS	$\mathcal{B}(D^+ \to f_0(1370)\pi^+)\mathcal{B}(f_0(1370) \to K^+K^-) = (6.2 \pm 1.1) \times 10^{-4}$	
PDG06	$\mathcal{B}(D^0 \to \sigma\pi^0)\mathcal{B}(f_0 \to \pi^+\pi^-) < 2.7 \times 10^{-5}$	$\mathcal{B}(D^0 \to \sigma\pi^0) < 4.1 \times 10^{-5}$
PDG06	$\mathcal{B}(D^0 \to f_0\pi^0)\mathcal{B}(f_0 \to \pi^+\pi^-) < 3.4 \times 10^{-6}$	$\mathcal{B}(D^0 \to f_0\pi^0) < 6.4 \times 10^{-6}$
PDG06	$\mathcal{B}(D^0 \to f_0K_s^0)\mathcal{B}(f_0 \to \pi^+\pi^-) = (1.36^{+0.30}_{-0.22}) \times 10^{-3}$	$\mathcal{B}(D^0 \to f_0K_s^0) = (2.6^{+0.7}_{-0.6}) \times 10^{-3}$
ARGUS,E687	$\mathcal{B}(D^0 \to f_0\overline{K}^0)\mathcal{B}(f_0 \to \pi^+\pi^-) = (3.2 \pm 0.9) \times 10^{-3}$	$\mathcal{B}(D^0 \to f_0\overline{K}^0) = (6.0 \pm 2.0) \times 10^{-3}$
CLEO	$\mathcal{B}(D^0 \to f_0\overline{K}^0)\mathcal{B}(f_0 \to \pi^+\pi^-) = (2.5^{+0.8}_{-0.5}) \times 10^{-3}$	$\mathcal{B}(D^0 \to f_0\overline{K}^0) = (4.7^{+1.7}_{-1.2}) \times 10^{-3}$
PDG06	$\mathcal{B}(D^0 \to f_0K_s^0)\mathcal{B}(f_0 \to K^+K^-) < 1.0 \times 10^{-4}$	$\mathcal{B}(D^0 \to f_0K_s^0) < 1.3 \times 10^{-3}$
BaBar	$\mathcal{B}(D^0 \to f_0\overline{K}^0)\mathcal{B}(f_0 \to K^+K^-) = (1.2 \pm 0.9) \times 10^{-3}$	$\mathcal{B}(D^0 \to f_0\overline{K}^0) = (1.5 \pm 1.1)\%$
PDG06	$\mathcal{B}(D^0 \to a_0^+K^-)\mathcal{B}(a_0^+ \to K^+K_s^0) = (6.1 \pm 1.8) \times 10^{-4}$	$\mathcal{B}(D^0 \to a_0^+K^-) = (7.9 \pm 2.5)^{-3}$
BaBar	$\mathcal{B}(D^0 \to a_0^+K^-)\mathcal{B}(a_0^+ \to K^+\overline{K}^0) = (3.3 \pm 0.8) \times 10^{-3}$	$\mathcal{B}(D^0 \to a_0^+K^-) = (2.1 \pm 0.6)\%$
PDG06	$\mathcal{B}(D^0 \to a_0^-K^+)\mathcal{B}(a_0^- \to K^-K_s^0) < 1.1 \times 10^{-4}$	$\mathcal{B}(D^0 \to a_0^-K^+) < 1.4 \times 10^{-3}$
BaBar	$\mathcal{B}(D^0 \to a_0^-K^+)\mathcal{B}(a_0^- \to K^-\overline{K}^0) = (3.1 \pm 1.9) \times 10^{-4}$	$\mathcal{B}(D^0 \to a_0^-K^+) = (2.0 \pm 1.2) \times 10^{-3}$
PDG06	$\mathcal{B}(D^0 \to a_0^0K_s^0)\mathcal{B}(a_0^0 \to K^+K^-) = (3.0 \pm 0.4) \times 10^{-3}$	$\mathcal{B}(D^0 \to a_0^0K_s^0) = (3.8 \pm 0.7)\%$
BaBar	$\mathcal{B}(D^0 \to a_0^0\overline{K}^0)\mathcal{B}(a_0^0 \to K^+K^-) = (5.9 \pm 1.3) \times 10^{-3}$	$\mathcal{B}(D^0 \to a_0^0\overline{K}^0) = (7.6 \pm 1.9)\%$
BaBar	$\mathcal{B}(D^0 \to a_0^+\pi^-)\mathcal{B}(a_0^+ \to K^+\overline{K}^0) = (5.1 \pm 4.2) \times 10^{-4}$	$\mathcal{B}(D^0 \to a_0^+\pi^-) = (3.3 \pm 2.7) \times 10^{-3}$
BaBar	$\mathcal{B}(D^0 \to a_0^-\pi^+)\mathcal{B}(a_0^- \to K^-K^0) = (1.43 \pm 1.19) \times 10^{-4}$	$\mathcal{B}(D^0 \to a_0^-\pi^+) = (9.2 \pm 7.7) \times 10^{-4}$
PDG06	$\mathcal{B}(D^0 \to K_0^{*-}\pi^+)\mathcal{B}(K_0^{*-} \to K_s^0\pi^-) = (2.8^{+0.6}_{-0.4}) \times 10^{-3}$	$\mathcal{B}(D^0 \to K_0^{*-}\pi^+) = (9.0^{+2.3}_{-1.7})^{-3}$
ARGUS,E687	$\mathcal{B}(D^0 \to K_0^{*-}\pi^+)\mathcal{B}(K_0^{*-} \to \overline{K}^0\pi^-) = (7.3 \pm 1.6) \times 10^{-3}$	$\mathcal{B}(D^0 \to K_0^{*-}\pi^+) = (1.2 \pm 0.3)\%$
CLEO	$\mathcal{B}(D^0 \to K_0^{*-}\pi^+)\mathcal{B}(K_0^{*-} \to \overline{K}^0\pi^-) = (4.3^{+1.9}_{-0.8}) \times 10^{-3}$	$\mathcal{B}(D^0 \to K_0^{*-}\pi^+) = (7.0^{+3.2}_{-1.5}) \times 10^{-3}$
PDG06	$\mathcal{B}(D^0 \to K_0^{*-}\pi^+)\mathcal{B}(K_0^{*-} \to K^-\pi^0) = (4.6 \pm 2.2) \times 10^{-3}$	$\mathcal{B}(D^0 \to K_0^{*-}\pi^+) = (1.5 \pm 0.7)\%$
CLEO	$\mathcal{B}(D^0 \to K_0^{*-}\pi^+)\mathcal{B}(K_0^{*-} \to K^-\pi^0) = (3.6 \pm 0.8) \times 10^{-3}$	$\mathcal{B}(D^0 \to K_0^{*-}\pi^+) = (1.2 \pm 0.3)\%$
PDG06	$\mathcal{B}(D^0 \to \overline{K}_0^{*0}\pi^0)\mathcal{B}(\overline{K}_0^{*0} \to K^-\pi^+) = (5.8^{+4.6}_{-1.5}) \times 10^{-3}$	$\mathcal{B}(D^0 \to \overline{K}_0^{*0}\pi^0) = (9.4^{+7.5}_{-2.6}) \times 10^{-3}$
CLEO	$\mathcal{B}(D^0 \to \overline{K}_0^{*0}\pi^0)\mathcal{B}(\overline{K}_0^{*0} \to K^-\pi^+) = (5.3^{+4.2}_{-1.4}) \times 10^{-3}$	$\mathcal{B}(D^0 \to \overline{K}_0^{*0}\pi^0) = (8.5^{+6.8}_{-2.5}) \times 10^{-3}$
PDG06	$\mathcal{B}(D^0 \to f_0(1370)K_s^0)\mathcal{B}(f_0(1370) \to \pi^+\pi^-) = (2.5 \pm 0.6) \times 10^{-3}$	
PDG06	$\mathcal{B}(D^0 \to a_0K_s^0)\mathcal{B}(a_0 \to \eta\pi^0) = (6.2 \pm 2.0) \times 10^{-3}$	
ARGUS,E687	$\mathcal{B}(D^0 \to f_0(1370)\overline{K}^0)\mathcal{B}(f_0(1370) \to \pi^+\pi^-) = (4.7 \pm 1.4) \times 10^{-3}$	
CLEO	$\mathcal{B}(D^0 \to f_0(1370)\overline{K}^0)\mathcal{B}(f_0(1370) \to \pi^+\pi^-) = (5.9^{+1.8}_{-2.7}) \times 10^{-3}$	
PDG06	$\mathcal{B}(D^0 \to f_0(1400)K_s^0)\mathcal{B}(f_0(1400) \to K^+K^-) = (1.7 \pm 1.1) \times 10^{-4}$	
PDG06	$\mathcal{B}(D_s^+ \to f_0\pi^+)\mathcal{B}(f_0 \to K^+K^-) = (5.7 \pm 2.5) \times 10^{-3}$	$\mathcal{B}(D_s^+ \to f_0\pi^+) = (7.1 \pm 3.2)\%$
E687	$\mathcal{B}(D_s^+ \to f_0\pi^+)\mathcal{B}(f_0 \to K^+K^-) = (4.9 \pm 2.3) \times 10^{-3}$	$\mathcal{B}(D_s^+ \to f_0\pi^+) = (6.1 \pm 3.0)\%$
E791	$\mathcal{B}(D_s^+ \to f_0\pi^+)\mathcal{B}(f_0 \to \pi^+\pi^-) = (5.7 \pm 1.7) \times 10^{-3}$	$\mathcal{B}(D_s^+ \to f_0\pi^+) = (1.1 \pm 0.4)\%$
FOCUS	$\mathcal{B}(D_s^+ \to f_0\pi^+)\mathcal{B}(f_0 \to \pi^+\pi^-) = (9.5 \pm 2.7) \times 10^{-3}$	$\mathcal{B}(D_s^+ \to f_0\pi^+) = (1.8 \pm 0.6)\%$
FOCUS	$\mathcal{B}(D_s^+ \to f_0\pi^+)\mathcal{B}(f_0 \to K^+K^-) = (7.0 \pm 1.9) \times 10^{-3}$	$\mathcal{B}(D_s^+ \to f_0\pi^+) = (8.8 \pm 2.6)\%$
FOCUS	$\mathcal{B}(D_s^+ \to f_0K^+)\mathcal{B}(f_0 \to K^+K^-) = (2.8 \pm 1.3) \times 10^{-4}$	$\mathcal{B}(D_s^+ \to f_0K^+) = (3.5 \pm 1.7) \times 10^{-3}$
PDG06	$\mathcal{B}(D_s^+ \to \overline{K}_0^{*0}K^+)\mathcal{B}(\overline{K}_0^{*0} \to K^-\pi^+) = (4.8 \pm 2.5) \times 10^{-3}$	$\mathcal{B}(D_s^+ \to \overline{K}_0^{*0}K^+) = (7.7 \pm 4.1) \times 10^{-3}$
E687	$\mathcal{B}(D_s^+ \to \overline{K}_0^{*0}K^+)\mathcal{B}(\overline{K}_0^{*0} \to K^-\pi^+) = (4.3 \pm 2.5) \times 10^{-3}$	$\mathcal{B}(D_s^+ \to \overline{K}_0^{*0}K^+) = (6.9 \pm 4.1) \times 10^{-3}$
FOCUS	$\mathcal{B}(D_s^+ \to K_0^{*0}\pi^+)\mathcal{B}(K_0^{*0} \to K^+\pi^-) = (1.4 \pm 0.8) \times 10^{-3}$	$\mathcal{B}(D_s^+ \to K_0^{*0}\pi^+) = (2.3 \pm 1.3) \times 10^{-3}$
E791	$\mathcal{B}(D_s^+ \to f_0(1370)\pi^+)\mathcal{B}(f_0(1370) \to \pi^+\pi^-) = (3.3 \pm 1.2) \times 10^{-3}$	

More precise measurements of these branching ratios are of great importance for studies of charmed meson to scalar meson transitions.

The theoretical studies of $D \to SP$ are very similar to $D \to PP$ except for the fact that the quark structure of the scalar mesons, especially $f_0(980)$ and $a_0(980)$, is still not clear (for a review, one can refer to [37] and references therein). Hence, one is facing a 'Scylla and Charybdis' dilemma at the moment when thinking to the limited theoretical control on the significant nonfactorized contributions as well. Yet it is without doubt that the study of charmed meson decays will open a new avenue to the understanding of the light scalar meson spectroscopy. One might resort to the knowledge resulting from $D \to PP$ and $D \to PV$ modes and try to make new understanding on some old puzzles related to the internal structure and parameters, e.g. the masses and widths, of light scalar mesons through the study of $D \to SP$ [38, 39, 40]. Or vice versa, one can start from the assumed structure of scalar mesons and make some predictions on the decays [41, 42, 43]. Some of the theoretical results in the literature are given in Table 4. In either case, the following factorization formula are useful.

$$T_S = -\frac{G_F}{\sqrt{2}} V_{q_1q_2} V^*_{cq_3} a_1 f_{_P} (m^2_{_D} - m^2_{_S}) F_0^{D\to S}(m^2_{_P}), \tag{44}$$

$$T_P = -\frac{G_F}{\sqrt{2}} V_{q_1q_2} V^*_{cq_3} a_1 f_{_S} (m^2_{_D} - m^2_{_P}) F_0^{D\to P}(m^2_{_S}), \tag{45}$$

$$C_S = -\frac{G_F}{\sqrt{2}} V_{q_1q_2} V^*_{cq_3} a_2 f_{_P} (m^2_{_D} - m^2_{_S}) F_0^{D\to S}(m^2_{_P}), \tag{46}$$

$$C_P = -\frac{G_F}{\sqrt{2}} V_{q_1q_2} V^*_{cq_3} a_2 f_{_S} (m^2_{_D} - m^2_{_P}) F_0^{D\to P}(m^2_{_S}). \tag{47}$$

3.4 $D \to AP$ Decays

There are two different types of axial vector mesons: ${}^3P_1$ and ${}^1P_1$, which carry the quantum numbers $J^{\rm PC} = 1^{++}$ and 1^{+-}, respectively. The isovector non-strange axial vector mesons $a_1(1260)$ and $b_1(1235)$ which correspond to ${}^3P_1$ and ${}^1P_1$, respectively, cannot have mixing because of the opposite C-parities. However, the isodoublet strange ones $K_1(1270)$ and $K_1(1400)$ are a mixture of ${}^3P_1$ and ${}^1P_1$ states due to the strange and non-strange light quark mass difference. One can usually write

$$\begin{aligned} K_1(1270) &= K_{1A}\sin\theta + K_{1B}\cos\theta \ , \\ K_1(1400) &= K_{1A}\cos\theta - K_{1B}\sin\theta \ , \end{aligned} \tag{48}$$

where K_{1A} and K_{1B} are the strange partners of $a_1(1260)$ and $b_1(1235)$ respectively.

Two-body hadronic $D \to AP$ decays have been studied in [44, 45, 46, 47, 48, 49, 50]. Under the factorization approximation, the decay amplitudes read

$$T_A = 2\frac{G_F}{\sqrt{2}} V_{q_1q_2} V^*_{cq_3} a_1 f_{_P} m_{_A} (\varepsilon^* \cdot p_{_D}) A_0^{D\to A}(m^2_{_P}), \tag{49}$$

Table 4: *Branching ratios for various $D \to SP$ decays. Experimental results are taken from Table 3.*

Decay	*Buccella etal.*[41]	*Cheng* [40]	Experiments
$D^+ \to f_0\pi^+$	2.8×10^{-4}	3.5×10^{-4}	$(3.6\pm1.1)\times10^{-4}$
$\to f_0K^+$	2.3×10^{-5}	2.2×10^{-5}	$\sim 10^{-4}$
$\to a_0^+\overline{K}^0$	6.4×10^{-3}	1.7×10^{-2}	
$\to a_0^0\pi^+$	5.9×10^{-4}	1.7×10^{-3}	$(3.1\pm0.7)\%$
$\to \sigma\pi^+$		input	$(2.1\pm0.5)\times10^{-3}$
$\to \kappa\pi^+$		input	$(6.5\pm1.9)\%$
$\to \overline{K}_0^{*0}\pi^+$		input	$(1.8\pm0.3)\%$
$\to a_0^0\pi^+$	5.9×10^{-4}		
$\to a_0^+\pi^0$	1.2×10^{-4}		
$\to a_0^+\eta$	7.4×10^{-4}		
$\to a_0^0K^+$	6.2×10^{-5}		
$\to f_0K^+$	2.3×10^{-5}		
$D^0 \to f_0\overline{K}^0$	7.4×10^{-4}	input	$\sim 10^{-3}-10^{-2}$
$\to a_0^+K^-$	7.8×10^{-4}	1.1×10^{-3}	$(2.1\pm0.6)\%$
$\to a_0^0\overline{K}^0$	2.2×10^{-3}	3.6×10^{-3}	$(7.6\pm1.9)\%$
$\to a_0^-K^+$	4.0×10^{-5}	7.9×10^{-5}	$(2.0\pm1.2)\times10^{-3}$
$\to a_0^+\pi^-$	3.0×10^{-5}	6.5×10^{-5}	$(3.3\pm2.7)\times10^{-3}$
$\to a_0^-\pi^+$	7.0×10^{-4}	1.3×10^{-3}	$(9.2\pm7.7)\times10^{-4}$
$\to K_0^{*-}\pi^+$		1.1×10^{-2}	$\sim 10^{-3}-10^{-2}$
$\to \overline{K}_0^{*0}\pi^0$		3.7×10^{-3}	$(8.5^{+6.8}_{-2.5})\times10^{-3}$
$\to f_0\pi^0$	6.0×10^{-6}		
$\to f_0\eta$	4.0×10^{-5}		
$\to a_0^0\pi^0$	1.1×10^{-4}		
$\to a_0^0\eta$	1.5×10^{-4}		
$D_s^+ \to f_0\pi^+$	1.1%	input	$(1.8\pm0.6)\%$
$\to f_0K^+$	6.9×10^{-4}	1.2×10^{-3}	$(3.5\pm1.7)\times10^{-3}$
$\to \overline{K}_0^{*0}K^+$		1.5×10^{-3}	$(6.9\pm4.1)\times10^{-3}$
$\to K_0^{*0}\pi^+$		1.1×10^{-3}	$(2.3\pm1.3)\times10^{-3}$
$\to a_0^+K^0$	3.0×10^{-5}		
$\to a_0^0K^+$	7.0×10^{-5}		
$\to a_0^+\eta$	7.0×10^{-5}		

$$T_P = 2\frac{G_F}{\sqrt{2}}V_{q_1q_2}V^*_{cq_3}a_1 f_A m_A(\varepsilon^*\cdot p_D)F_+^{D\to P}(m_A^2), \tag{50}$$

$$C_A = 2\frac{G_F}{\sqrt{2}}V_{q_1q_2}V^*_{cq_3}a_2 f_P m_A(\varepsilon^*\cdot p_D)A_0^{D\to A}(m_P^2), \tag{51}$$

$$C_P = 2\frac{G_F}{\sqrt{2}}V_{q_1q_2}V^*_{cq_3}a_2 f_A m_A(\varepsilon^*\cdot p_D)F_+^{D\to P}(m_A^2). \tag{52}$$

The predicted branching ratios from factorizable contributions for $D^0 \to K^- a_1^+$ and $D^0 \to K_1^-(1270)\pi^+$ and that for $D^0 \to \overline{K}^0 a_1^+$ and $D^+ \to \overline{K}_1^0(1400)\pi^+$ in Refs. [44, 45, 46, 47, 48] were too small by roughly a factor of 5 and 2, respectively, when compared with experimental data. One argument is that the factorization approach may be only suitable for energetic two-body decays like $D \to PP$ and $D \to PV$; for $D \to AP$ with very little energy release, the approximation is questionable since the nonperturbative contributions grow up. A recent analysis [50] considering the sizable FSI effects showed that the predictions, which are presented in Table 5 and Table 6, are improved greatly.

Table 5: *Branching ratios for $D \to Ka_1(1260)$ and $D \to Kb_1(1235)$. Most decay modes involving a neutral K meson are given as K_S^0 in PDG06 instead of $\overline{K}^0$ in PDG04 which are presented here as well.*

Decay	Theory [50]		Experiment [9]
	without FSIs	with FSIs	
$D^+ \to \overline{K}^0 a_1^+(1260)$	12.1%	12.1%	$(3.6 \pm 0.6)\%$
			$(8.2 \pm 1.7)\%$ (*pdg*04)
$D^0 \to K^- a_1^+(1260)$	3.8%	6.2%	$(7.5 \pm 1.1)\%$
$D^0 \to \overline{K}^0 a_1^0(1260)$	3.3×10^{-4}	5.6×10^{-4}	$< 1.9\%$
$D^+ \to \overline{K}^0 b_1^+(1235)$	1.7×10^{-3}	1.7×10^{-3}	
$D^0 \to K^- b_1^+(1235)$	3.7×10^{-6}	5.9×10^{-6}	
$D^0 \to \overline{K}^0 b_1^0(1235)$	3.9×10^{-4}	6.7×10^{-4}	

3.5 $D \to TP$ Decays

The observed $J^P = 2^+$ tensor mesons $f_2(1270)$, $f_2'(1525)$, $a_2(1320)$ and $K_2^*(1430)$ form an SU(3) $1\,^3P_2$ nonet with quark content $q\bar{q}$. Hadronic charm decaying to a pseudoscalar meson and a tensor meson $f_2(1270)$, $a_2(1320)$ or $K_2^*(1430)$ have been found in the earlier measurements by ARGUS [29] and E687 [31], and in recent experiments from E791 [33], CLEO [30], FOCUS [34] and BaBar [35], though some of them have not yet sufficient statistical significance. The results of various experiments are summarized in Table 7 where the products of $\mathcal{B}(D \to TP_3)$ and $\mathcal{B}(T \to P_1P_2)$ are shown. It is evident that most of the listed $D \to TP$ decays have branching ratios of order 10^{-3}, even though some of them are Cabibbo suppressed. In order to extract the branching ratios for $D \to TP$ decays, one must use the branching fractions of the strong decays of the tensor mesons [9]:

$$\begin{array}{ll} \mathcal{B}(f_2(1270) \to \pi\pi) = (84.7^{+2.5}_{-1.2})\%, & \mathcal{B}(f_2(1270) \to K\overline{K}) = (4.6 \pm 0.4)\%, \\ \mathcal{B}(a_2(1320) \to K\overline{K}) = (4.9 \pm 0.8)\%, & \mathcal{B}(K_2^*(1430) \to K\pi) = (49.9 \pm 1.2)\% \end{array} \quad (53)$$

Theoretical calculations based on the factorization hypothesis were conducted to understand the experimental data [51, 52, 53]. Some of the results are listed in Table 8 where one can find that most of the theoretical predictions are not consistent with the experimental data. At first glance, some decays like $D \to \overline{K}_2^*(1430)K$ and $D^0 \to f_2'(1525)\overline{K}^0$ *etal.*, are kinematically not allowed as the total mass of the final state particles lies outside of the phase space for the decay. Nevertheless, it is physically allowed as some tensor mesons have broad widths of order several hundred MeV [9].

3.6 Other Decay Modes

Measurements of other nonleptonic two-body modes, such as $D \to AV$ and so on , have been done in experiment. PDG report two fairly large branching ratios [9]: $\mathcal{B}(D^+ \to \overline{K}^{*0}a_1(1260)^+) = (9.4 \pm 1.9) \times 10^{-3}$ and $\mathcal{B}(D_s^+ \to \phi a_1(1260)^+) = (2.9 \pm 0.7)\%$, though the total masses of their final state mesons exceed their phase spaces. Given the relevant form factors, the branching fractions can be worked out by means of factorization approach. Yet one doubt that how to evaluate the reliability of factorization in these decay modes will arise since the nonfactorized corrections may be dominant due to the small momentum transfer.

4 Three-Body Decays

4.1 Kinematics and Dalitz Plot

Starting from Equation (24) and integrating over the solid angles, the decay rate for $D \to M_1M_2M_3$ can be obtained

$$d\Gamma = \frac{1}{(2\pi)^3}\frac{1}{32m_D^3}|\mathcal{A}|^2 dm_{12}^2 dm_{23}^2 \ , \quad (54)$$

where m_{ij} is the invariant mass of particles i and j. For a given value of m_{12}^2 in the range $(m_1+m_2)^2 \leq m_{12}^2 \leq (m_D - m_3)^2$, the upper and lower bounds of m_{23}^2 are determined

$$(m_{23}^2)_{max} = (E_2^* + E_3^*)^2 - (\sqrt{E_2^{*2} - m_2^2} - \sqrt{E_3^{*2} - m_3^2})^2 \ , \quad (55)$$

$$(m_{23}^2)_{max} = (E_2^* + E_3^*)^2 - (\sqrt{E_2^{*2} - m_2^2} + \sqrt{E_3^{*2} - m_3^2})^2 \ . \quad (56)$$

Table 6: *Branching ratios of $D \to K_1(1270)\pi$ and $D \to K_1(1400)\pi$ calculated for various $K_{1A} - K_{1B}$ mixing angles.*

Decay	Theory [50]				Experiment [9]
	$-37°$	$-58°$	$37°$	$58°$	
$D^+ \to \overline{K}_1^0(1270)\pi^+$	6.4×10^{-3}	7.8×10^{-3}	2.9%	4.7%	$< 7 \times 10^{-3}$
$D^+ \to \overline{K}_1^0(1400)\pi^+$	2.9%	4.0%	6.6%	6.6%	$(4.3 \pm 1.5)\%$
$D^0 \to K_1^-(1270)\pi^+$	6.3×10^{-3}	5.5×10^{-3}	4.9×10^{-4}	4.4×10^{-5}	$(1.12 \pm 0.31)\%$
$D^0 \to K_1^-(1400)\pi^+$	3.7×10^{-8}	4.2×10^{-4}	3.0×10^{-3}	3.2×10^{-3}	$< 1.2\%$
$D^0 \to \overline{K}_1^0(1270)\pi^0$	8.4×10^{-3}	8.4×10^{-3}	8.4×10^{-3}	8.4×10^{-3}	
$D^0 \to \overline{K}_1^0(1400)\pi^0$	5.7×10^{-3}	5.5×10^{-3}	5.7×10^{-3}	5.5×10^{-3}	$< 3.7\%$

Table 7: *Experimental branching ratios of various $D \to TP$ decays measured by BaBar, CLEO, E791, FOCUS and PDG06. For simplicity and convenience, we have dropped the mass identification for $f_2(1270)$, $a_2(1320)$ and $K_2^*(1430)$.*

Collaboration	$\mathcal{B}(D \to TP) \times \mathcal{B}(T \to P_1P_2)$	$\mathcal{B}(D \to TP)$
PDG06	$\mathcal{B}(D^+ \to f_2\pi^+)\mathcal{B}(f_2 \to \pi^+\pi^-) = (4.8 \pm 1.3) \times 10^{-4}$	$\mathcal{B}(D^+ \to f_2\pi^+) = (8.5 \pm 2.3) \times 10^{-4}$
E791	$\mathcal{B}(D^+ \to f_2\pi^+)\mathcal{B}(f_2 \to \pi^+\pi^-) = (6.0 \pm 1.1) \times 10^{-4}$	$\mathcal{B}(D^+ \to f_2\pi^+) = (1.1 \pm 0.2) \times 10^{-3}$
FOCUS	$\mathcal{B}(D^+ \to f_2\pi^+)\mathcal{B}(f_2 \to \pi^+\pi^-) = (3.8 \pm 0.8) \times 10^{-5}$	$\mathcal{B}(D^+ \to f_2\pi^+) = (6.8 \pm 1.4) \times 10^{-4}$
FOCUS	$\mathcal{B}(D^+ \to f_2\pi^+)\mathcal{B}(f_2 \to K^+K^-) = (7.0 \pm 1.9) \times 10^{-5}$	$\mathcal{B}(D^+ \to f_2\pi^+) = (3.1 \pm 0.9) \times 10^{-3}$
PDG06	$\mathcal{B}(D^+ \to K_2^{*0}\pi^+)\mathcal{B}(K_2^{*0} \to K^+\pi^-) = (5.2 \pm 3.5) \times 10^{-5}$	$\mathcal{B}(D^+ \to K_2^{*0}\pi^+) = (1.6 \pm 1.1) \times 10^{-4}$
E791	$\mathcal{B}(D^+ \to \overline{K}_2^{*0}\pi^+)\mathcal{B}(\overline{K}_2^{*0} \to K^-\pi^+) = (4.6 \pm 2.0) \times 10^{-4}$	$\mathcal{B}(D^+ \to \overline{K}_2^{*0}\pi^+) = (1.4 \pm 0.6) \times 10^{-3}$
PDG06		$\mathcal{B}(D^+ \to a_2^+ K_S^0) < 1.5 \times 10^{-3}$
PDG06	$\mathcal{B}(D^0 \to f_2 K_S^0)\mathcal{B}(f_2 \to \pi^+\pi^-) = (1.3^{+1.1}_{-0.7}) \times 10^{-4}$	$\mathcal{B}(D^0 \to f_2 K_S^0) = (2.3^{+2.0}_{-1.3}) \times 10^{-4}$
CLEO	$\mathcal{B}(D^0 \to f_2\overline{K}^0)\mathcal{B}(f_2 \to \pi^+\pi^-) = (1.6^{+2.4}_{-1.3}) \times 10^{-4}$	$\mathcal{B}(D^0 \to f_2\overline{K}^0) = (2.8^{+4.3}_{-2.3}) \times 10^{-3}$
BaBar	$\mathcal{B}(D^0 \to a_2^-\pi^+)\mathcal{B}(a_2^- \to K^0K^-) = (3.5 \pm 2.1) \times 10^{-5}$	$\mathcal{B}(D^0 \to a_2^-\pi^+) = (7.0 \pm 4.3) \times 10^{-4}$
PDG06	$\mathcal{B}(D^0 \to K_2^{*-}\pi^+)\mathcal{B}(K_2^{*-} \to K_S^0\pi^-) = (3.2^{+2.1}_{-1.1}) \times 10^{-4}$	$\mathcal{B}(D^0 \to K_2^{*-}\pi^+) = (2.0^{+1.3}_{-0.7}) \times 10^{-3}$
CLEO	$\mathcal{B}(D^0 \to K_2^{*-}\pi^+)\mathcal{B}(K_2^{*-} \to \overline{K}^0\pi^-) = (6.5^{+4.2}_{-2.2}) \times 10^{-4}$	$\mathcal{B}(D^0 \to K_2^{*-}\pi^+) = (2.0^{+1.3}_{-0.7}) \times 10^{-3}$
BaBar	$\mathcal{B}(D^0 \to K_2^{*+}K^-)\mathcal{B}(K_2^{*+} \to K^0\pi^+) = (6.8 \pm 4.2) \times 10^{-4}$	$\mathcal{B}(D^0 \to K_2^{*+}K^-) = (2.0 \pm 1.3) \times 10^{-3}$
BaBar	$\mathcal{B}(D^0 \to \overline{K}_2^{*0}K^0)\mathcal{B}(\overline{K}_2^{*0} \to K^-\pi^+) = (6.6 \pm 2.7) \times 10^{-4}$	$\mathcal{B}(D^0 \to \overline{K}_2^{*0}K^0) = (2.0 \pm 0.8) \times 10^{-3}$
PDG06		$\mathcal{B}(D^0 \to a_2^+K^-) < 2 \times 10^{-3}$
PDG06	$\mathcal{B}(D_s^+ \to f_2\pi^+)\mathcal{B}(f_2 \to \pi^+\pi^-) = (1.2 \pm 0.7) \times 10^{-3}$	$\mathcal{B}(D_s^+ \to f_2\pi^+) = (2.1 \pm 1.3) \times 10^{-3}$
E791	$\mathcal{B}(D_s^+ \to f_2\pi^+)\mathcal{B}(f_2 \to \pi^+\pi^-) = (2.0 \pm 0.7) \times 10^{-3}$	$\mathcal{B}(D_s^+ \to f_2\pi^+) = (3.5 \pm 1.2) \times 10^{-3}$
FOCUS	$\mathcal{B}(D_s^+ \to f_2\pi^+)\mathcal{B}(f_2 \to \pi^+\pi^-) = (1.0 \pm 0.3) \times 10^{-3}$	$\mathcal{B}(D_s^+ \to f_2\pi^+) = (1.8 \pm 0.5) \times 10^{-3}$
FOCUS	$\mathcal{B}(D_s^+ \to f_2K^+)\mathcal{B}(f_2 \to \pi^+\pi^-) = (2.0 \pm 1.3) \times 10^{-4}$	$\mathcal{B}(D_s^+ \to f_2K^+) = (3.5 \pm 2.3) \times 10^{-4}$

Table 8: *Branching ratios for various $D \to TP$ decays. Experimental results are taken from Table 7.*

Decay	*Katoch etal.*[51]	*Muñoz etal.*[52]	*Cheng*[53] without FSIs	*Cheng*[53] with FSIs	Experiment
$D^+ \to f_2(1270)\pi^+$		7.97×10^{-6}	2.9×10^{-5}	2.2×10^{-4}	$(0.9 \pm 0.1) \times 10^{-3}$
$D^0 \to f_2(1270)\pi^0$		2.47×10^{-7}			
$D^0 \to f_2(1270)\overline{K}^0$	9.0×10^{-5}		1.0×10^{-4}	2.5×10^{-4}	$(4.5 \pm 1.7) \times 10^{-3}$
$D_s^+ \to f_2(1270)\pi^+$	3.6×10^{-4}		6.6×10^{-5}	2.1×10^{-3}	$(2.1 \pm 0.5) \times 10^{-3}$
$\to f_2(1270)K^+$			5.2×10^{-6}	4.9×10^{-5}	$(3.5 \pm 2.3) \times 10^{-4}$
$D^+ \to f_2'(1525)\pi^+$		7.18×10^{-9}	1.4×10^{-6}	3.7×10^{-6}	
$D^0 \to f_2'(1525)\pi^0$		2.18×10^{-10}			
$D^0 \to f_2'(1525)\overline{K}^0$			2.5×10^{-7}	6.0×10^{-7}	
$D_s^+ \to f_2'(1525)\pi^+$	1.3×10^{-2}		1.6×10^{-4}	1.5×10^{-4}	
$\to f_2'(1525)K^+$			4.9×10^{-6}	7.5×10^{-6}	
$D^+ \to a_2^+(1320)\pi^0$		9.05×10^{-7}			
$D^+ \to a_2^0(1320)\pi^+$		5.55×10^{-6}			
$D^+ \to a_2^+(1320)\overline{K}^0$	1.1×10^{-4}		1.3×10^{-6}	1.3×10^{-6}	$< 3 \times 10^{-3}$
$D^0 \to a_2^-(1320)\pi^+$		4.21×10^{-6}	5.7×10^{-6}	6.1×10^{-6}	$(7.0 \pm 4.3) \times 10^{-4}$
$\to a_2^0(1320)\pi^0$		1.72×10^{-7}			
$\to a_2^+(1320)K^-$	0		0	8.9×10^{-8}	$< 2 \times 10^{-3}$
$\to a_2^0(1320)\overline{K}^0$	1.7×10^{-5}				
$D^+ \to \overline{K}_2^{*0}(1430)\pi^+$	9.9×10^{-3}		2.6×10^{-4}	2.6×10^{-4}	$(1.4 \pm 0.6) \times 10^{-3}$
$D^0 \to K_2^{*-}(1430)\pi^+$	4.1×10^{-3}		1.0×10^{-4}	1.1×10^{-4}	$(2.0^{+1.3}_{-0.7}) \times 10^{-3}$
$\to \overline{K}_2^{*0}(1430)\pi^0$	0		0	1.3×10^{-5}	$< 3.4 \times 10^{-3}$
$\to K_2^{*+}(1430)K^-$			0	1.3×10^{-6}	$(2.0 \pm 1.3) \times 10^{-3}$
$\to \overline{K}_2^{*0}(1430)K^0$			0	$\sim 10^{-8}$	$(2.0 \pm 0.8) \times 10^{-3}$
$D_s^+ \to \overline{K}_2^{*0}(1430)K^+$	0				
$\to \overline{K}_2^{*+}(1430)\overline{K}^0$	4.2×10^{-5}				

E_2^* and E_3^* are the energies of final state mesons M_2 and M_3 respectively in the rest frame of M_1 and M_2

$$E_2^* = \frac{m_{12}^2 - m_1^2 + m_2^2}{2m_{12}} , \tag{57}$$

$$E_3^* = \frac{m_D^2 - m_{12}^2 - m_2^2}{2m_{12}} . \tag{58}$$

The scatter plot in m_{12}^2 versus M_{23}^2 is called a Dalitz Plot. For a detailed introduction of Dalitz technique, please refer to Chapter 4 and Ref. [54]. The amplitude $|\mathcal{A}|^2$ of a nonresonant decay is parameterized as constant without variation in magnitude or phase across the Dalitz plot. Therefore, the allowed region of the plot is uniformly populated with events. A nonuniformity with bands near the mass of the resonance in the plot will reflect the appearance of resonant contribution. One can find a review of Dalitz plot application on charm decays in [55].

4.2 Resonant Three-Body Decays

Charmed meson three-body decays proceed dominantly via quasi-two-body decays containing an intermediate resonance state which then decays to two particles. The analysis of these resonant decays using the Dalitz plot technique enables one to study the dynamical properties of various resonances. In theoretical studies, resonant decays are often divided into product of two processes: $\mathcal{B}(D \to RM_3) \times \mathcal{B}(R \to M_1M_2)$, just as we have shown in Section 3. Therefore what we confront with is in fact charmed meson two-body decays.

4.3 Nonresonant Three-Body Decays

The nonresonant contribution is usually a small fraction of the total 3-body decay rate. Experimentally, they are hard to be measured as the interference between nonresonant and quasi-two-body amplitudes makes it difficult to disentangle these two distinct contributions and extract the nonresonant one. Theoretically, the matrix element of D decaying to three mesons in general has two types of formalism in the factorization approximation, relying on how to distribute three final mesons into two "clusters".

For one type with a "cluster" where D transits to a light meson, one can find

$$\langle M_1M_2M_3|J_{i\mu}J_i'^{\mu}|D\rangle \sim \langle M_1M_2|J_{i\mu}|0\rangle\langle M_3|J_i'^{\mu}|D\rangle. \tag{59}$$

It is obvious that its contribution is negligibly small since matrix element $\langle M_1M_2|J_{i\mu}|0\rangle$ which also appears in the factorizable contributions of weak annihilation in two-body decays vanishes in the chiral limit.

For the other type with a "cluster" where D transits to two light mesons, the factorized formula reads

$$\langle M_1M_2M_3|J_{i\mu}J_i'^{\mu}|D\rangle \sim \langle M_1|J_{i\mu}|0\rangle\langle M_2M_3|J_i'^{\mu}|D\rangle. \tag{60}$$

Here a new matrix element $\langle M_2 M_3 | J_i'^{\mu} | D \rangle$ is introduced. It has the general expression [56]

$$\begin{aligned}\langle M_2(p_2) M_3(p_3) | J_i'^{\mu} | D(p_D) \rangle &= ir(p_D - p_2 - p_3)^{\mu} + i\omega_+(p_2 + p_3)^{\mu} + i\omega_-(p_3 - p_2)^{\mu} \\ &\quad + h\epsilon^{\mu\nu\alpha\beta} p_{D\nu}(p_2 + p_3)_{\alpha}(p_3 - p_2)_{\beta} \ , \end{aligned} \tag{61}$$

where r, $\omega_{\pm}$ and h are form factors. In general they receive two distinct contributions: one from the point-like weak transition and the other from the pole diagrams which involve four-point strong vertices. Models based on chiral symmetry and heavy quark effective theory have been developed to make some estimates on them [56, 57, 58].

Charmed meson to three pseudoscalar nonresonant decays have been studied in the approach of effective $SU(4)_L \times SU(4)_R$ chiral Lagrangian [59, 60, 61, 62, 63], in which the predictions of the branching ratios were in general too small when compared with experiment. With the advent of heavy meson chiral perturbation theory (HMChPT) [64, 65, 66], the nonresonant D decays can be studied reliably at least in the kinematical region where the final pseudoscalar mesons are soft [67, 68, 69]. Some theoretical results are collected in Table 9.

4.4 Beyond Three-Body Decays

Some multi-body charm meson decays, even up to seven-body, have been measured in experiment [9]. Yet our available theoretical tools lose much of their power when applied to genuine multi-body transitions. The kinematics structure and strong dynamics will be more and more complicated and ultimately out of control with the number of final state particles increase.

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Table 9: *Branching ratios (in %) for nonresonant 3-body D decays in various models. Most decay modes involving a neutral K meson are given as K_S^0 in PDG06 instead of $\overline{K}^0$ in PDG04 which are presented here as well.*

Decay mode	Chau *et al.* [62]	Botella *et al.* [63]	Cheng *et al.*[69]	PDG 06/04
$D^0 \to \overline{K}^0\pi^+\pi^-$	0.13	0.19	0.03 ; 0.17	$0.026^{+0.059}_{-0.016}$
				$0.054^{+0.120}_{-0.034}$ (*pdg*04)
$\to K^-\pi^+\pi^0$	0.18	0.76	0.61 ; 0.28	$1.13^{+0.54}_{-0.20}$
$\to \overline{K}^0K^+K^-$	0.02	0.006	0.16 ; 0.01	
$\to \pi^+\pi^-\pi^0$	0.04	0.11		
$\to K^+K^-\pi^0$		0.013		
$\to K^0K^-\pi^+$		0.007		0.11 ± 0.11
				0.23 ± 0.23 (*pdg*04)
$\to \overline{K}^0K^+\pi^-$		0.013		$0.19^{+0.11}_{-0.08}$
				$0.38^{+0.23}_{-0.19}$ (*pdg*04)
$\to \overline{K}^0\pi^0\pi^0$				0.42 ± 0.11
				0.85 ± 0.22 (*pdg*04)
$D^+ \to \overline{K}^0\pi^+\pi^0$	0.76	1.9	1.5;0.7	0.9 ± 0.7
				1.3 ± 1.1 (*pdg*04)
$\to K^-\pi^+\pi^+$	1.71	0.95	6.5 ; 1.6	9.0 ± 0.7
$\to \pi^+\pi^+\pi^-$	0.15	0.19	0.50 ; 0.067	
$\to K^-K^+\pi^+$	0.02	0.016	0.48 ; 0.004	
$\to K^+\pi^+\pi^-$		0.0032		
$\to K^+K^+K^-$		1.58×10^{-5}		
$\to \pi^+\eta\eta$		0.016 ; 0.032		
$\to \pi^+\eta\eta'$		0.032		
$D_s^+ \to K^-K^+\pi^+$	0.42	0.32	1.0 ; 0.69	
$\to \pi^+\pi^+\pi^-$	5×10^{-5}	4.7×10^{-4}		
$\to \pi^+\pi^0\eta$		1.1 ; 0.95		< 5
$\to \pi^+\pi^0\eta'$		0.158		< 1.8
$\to K^+\pi^+\pi^-$		0.047		0.1 ± 0.04

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