Chapter 4

Analysis tools

4.1 Dalitz-plot Analysis Formalism[1]

Originally the primary application of Dalitz-plot analyses was to determine the spin and parity of light mesons. Recently Dalitz-plot analyses have emerged as a powerful tool in the study of D and B mesons.

Charm meson decay dynamics have been studied extensively over the last decade. Recent studies of multi-body decays of charm mesons probe a variety of physics including doubly-Cabibbo suppressed decays[1][2][3], searches for CP violation[2][4][5][6][7], T violation[8], D^0–$\overline{D}^0$ mixing[9][10], the properties of established light mesons[11][12][13][14], the properties of $\pi\pi$[1][13][15], $K\pi$[16][17], and KK [18] S-wave states, and the dynamics of four-body final states[19][20].

Recently B meson decay dynamics have been studied. Multi-body decays of B mesons also probe a variety of physics including, charmless B-decays[21] [**?**][23][24][25], measurements of the Cabibbo-Kobayashi-Maskawa (CKM) angle γ/ϕ_3 [26][27][28][29][30], searches for direct CP violation[23] [24][31], charm spectroscopy[32][33], the properties of established light mesons [21][24][25], the properties of KK[21][25] and $K\pi$[21][23][24] S-wave, and the three-body production of baryons[**?**][34]. Time-dependent Dalitz-plot (TD) analyses have been used to determine the CKM angle α/ϕ_2 with $B \to \pi^+\pi^-\pi^0$[35] and to resolve the two-fold ambiguity in the CKM angle β/ϕ_1 with $B \to D\pi^0, D \to K^0_S\pi^+\pi^-$[36][37]. A TD analysis of $B^0 \to D^{*\pm}K^0\pi^{\mp}$[38] is sensitive γ/ϕ_3. Future studies could improve sensitivity to new physics in TD analyses of $b \to s$ penguin decays[25].

Additionally, partial wave analyses have been used to study the dynamics of charmonium decays to hadrons, following the formalism presented in Refs.[39][40], in radiative decays[41][42][43][44] and in decays to all hadronic final states[45][46][47][48]. Multi-body decays of charmonium to all hadronic final states can be analyzed with the Dalitz-plot analysis technique. Studies of the $\pi\pi$, $K\pi$ and KK S-wave in charmonium decays probe most of the phase space accessible in B decays. Thus Dalitz-plot analyses of charmonia could lead to reduced systematic errors in many B analyses.

Weak nonleptonic decays of B and charm mesons are expected to proceed dominantly through resonant two-body decays in several theoretical models[49]; see Ref.[50] for a

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review of resonance phenomenology. These amplitudes are typically calculated with the Dalitz plot analysis technique[51], which uses the mininum number of independent observable quantities. For three-body decay of a spin-0 particle to all pseudo-scalar final states, $D, B \to abc$, the decay rate[52] is

$$\Gamma = \frac{1}{(2\pi)^3 32\sqrt{s^3}} |\mathcal{M}|^2 \, dm_{ab}^2 dm_{bc}^2, \tag{4.1}$$

where m_{ij} is the invariant mass of $i-j$ and the coefficient of the amplitude includes all kinematic factors. The scatter plot in m_{ab}^2 versus m_{bc}^2 is called a Dalitz plot. If $|\mathcal{M}|^2$ is constant the allowed region of the plot will be populated uniformly with events. Any variation in $|\mathcal{M}|^2$ over the Dalitz plot is due to dynamical rather than kinematical effects. It is straightforward to extend the formalism beyond three-body final states. For N-body final states, phase space has dimension $3N-7$. Other cases of interest include one vector particle or a fermion/anti-femion pair (e.g. $B \to D^*\pi\pi$, $B \to \Lambda_c p\pi$, $B \to K\ell\ell$) in the final state. For the former case phase space has dimension $3N-5$ and for the latter two $3N-4$.

4.2 Formalism[2]

The amplitude of the process, $R \to rc, r \to ab$ where R is a D, B, or $q\bar{q}$ meson and a,b,c are pseudo-scalars, is given by

$$\begin{aligned}\mathcal{M}_r(J,L,l,m_{ab},m_{bc}) &= \textstyle\sum_\lambda \langle ab|r_\lambda\rangle \, T_r(m_{ab}) \, \langle cr_\lambda|R_J\rangle \\ &= Z(J,L,l,\vec{p},\vec{q}) B_L^R(|\vec{p}|) B_L^r(|\vec{q}|) T_r(m_{ab}),\end{aligned} \tag{4.2}$$

where the sum is over the helicity states λ of the intermediate resonance particle r, a and b are the daughter particles of the resonance r, c is the spectator particle, J is the total angular momentum of R, L is the orbital angular momentum between r and c, l is the orbital angular momentum between a and b equivalent to the spin of r, $\vec{p}$ and $\vec{q}$ are the momentum of c and a, respectively, in the r rest frame, Z describes the angular distribution of final state particles, B_L^R and B_L^r are the barrier factors for the production of rc and ab, respectively, with angular momentum L, and T_r is the dynamical function describing the resonance r. The amplitude for modeling the Dalitz plot is a phenomenological object. Differences in the parametrizations of Z, B_L and T_r, as well as the set of resonances r, complicate the comparison of results from different experiments.

Usually the resonances are modeled with a Breit-Wigner although some more recent analyses have used the K-matrix formalism[53][54][55] with the P-vector approximation[56] to describe the $\pi\pi$ S-wave.

The nonresonant (NR) contribution $D \to abc$ is parameterized as constant (S-wave) with no variation in magnitude or phase across the Dalitz plot. The available phase space is much greater for B decay and the nonresonant contribution to $B \to abc$ requires a more sophisticated parametrization. Theoretical models of the NR amplitude[57][58] [59][60] do not reproduce the distibutions observed in the data. Experimentally, several parametrizations have been used[21][25].

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4.2.1 Barrier Factor B_L

The maximum angular momentum L in a strong decay is limited by the linear momentum $\vec{q}$. Decay particles moving slowly with an impact parameter (meson radius) d of order 1 fm have difficulty generating sufficient angular momentum to conserve the spin of the resonance. The Blatt-Weisskopf[61][62] functions B_L, given in **??**, weight the reaction amplitudes to account for this spin-dependent effect. These functions are normalized to give $B_L = 1$ for $z = (|\vec{q}|\, d)^2 = 1$. Another common formulation B'_L, also in **??**, is normalized to give $B'_L = 1$ for $z = z_0 = (|\vec{q}_0|\, d)^2$ where q_0 is the value of q when $m_{ab} = m_r$.

L	$B_L(q)$	$B'_L(q, q_0)$
0	1	1
1	$\sqrt{\frac{2z}{1+z}}$	$\sqrt{\frac{1+z_0}{1+z}}$
2	$\sqrt{\frac{13z^2}{(z-3)^2+9z}}$	$\sqrt{\frac{(z_0-3)^2+9z_0}{(z-3)^2+9z}}$
	where $z = (\vert\vec{q}\vert\, d)^2$ and $z_0 = (\vert\vec{q}_0\vert\, d)^2$	

Table 4.1: Blatt-Weisskopf barrier factors.

4.2.2 Angular Distributions

The tensor or Zemach formalism[63][64] and the helicity formalism[65][64] yield identical descriptions of the angular distributions for the decay process $R \to rc, r \to ab$ for reactions where a, b and c are spin-0 and the initial state is unpolarized. In this scenario, the angular distributions for $J = 0, 1, 2$ are given in 4.2. For polarized initial states, the helicity formalism[65] is used to determine the distinct angular distribution for each helicity state $|\lambda|$. The angular distributions for $J = 1, 2$ for a polarized initial are given in 4.3. The sign of the helicity cannot be determined from the Dalitz plot alone when a, b and c are spin-0. For final state particles with non-zero spin (e.g. radiative charmonium decays), the helicity formalism is required.

For the decays of pseudoscalars to three pseudoscalars the formalism simplifies considerably as the angular distribution Z depends only on the spin l of resonance r. Since $J = 0$ and $L = l$ only the first three rows of 4.2 are required.

4.2.3 Dynamical Function T_R

The dynamical function T_r is derived from the S-matrix formalism. In general, the amplitude that a final state f couples to an initial state i $S_{fi} = \langle f|S|i\rangle$, where the scattering operator S is unitary and satisfies $SS^\dagger = S^\dagger S = I$. The transition operator $\hat{T}$ is defined by separating the probability that $f = i$ yielding,

$$S = I + 2iT = I + 2i\,\{\rho\}^{1/2}\,\hat{T}\,\{\rho\}^{1/2}\,, \tag{4.3}$$

$J \to l + L$	Angular Distribution
$0 \to 0+0$	uniform
$0 \to 1+1$	$(1+\zeta^2)\cos^2\theta$
$0 \to 2+2$	$(\zeta^2+\frac{3}{2})^2(\cos^2\theta-1/3)^2$
$1 \to 0+1$	uniform
$1 \to 1+0$	$1+\zeta^2\cos^2\theta$
$1 \to 1+1$	$\sin^2\theta$
$1 \to 1+2$	$1+(3+4\zeta^2)\cos^2\theta$
$1 \to 2+1$	$(1+\zeta^2)[1+3\cos^2\theta+9\zeta^2(\cos^2\theta-1/3)^2]$
$1 \to 2+2$	$(1+\zeta^2)\cos^2\theta\sin^2\theta$
$2 \to 0+2$	uniform
$2 \to 1+1$	$3+(1+4\zeta^2)\cos^2\theta$
$2 \to 1+2$	$\sin^2\theta$
$2 \to 2+0$	$1+\frac{\zeta^2}{3}+\zeta^2\cos^2\theta+\zeta^4(\cos^2\theta-1/3)^2$
$2 \to 2+1$	$1+\frac{\zeta^2}{9}+(\frac{\zeta^2}{3}-1)\cos^2\theta-\zeta^2(\cos^2\theta-1/3)^2$
$2 \to 2+2$	$1+\frac{\zeta^2}{9}+(\frac{\zeta^2}{3}-1)\cos^2\theta+\frac{(16\zeta^4+21\zeta^2+9)(\cos^2\theta-1/3)^2}{3}$

Table 4.2: Angular distributions for each J, L, l for *unpolarized* initial states where θ is the angle between particles a and c in the rest frame of resonance r, $\sqrt{1+\zeta^2}$ is a relativistic correction with $\zeta^2 = E_r^2/m_{ab}^2 - 1$, and $E_r = (m_R^2 + m_{ab}^2 - m_c^2)/2m_R$.

$J \to l + L$	Angular Distribution
$1 \to 1+0$	$F_0\gamma^2\cos^2\theta + F_1\sin^2\theta$
$1 \to 1+1$	$F_1\sin^2\theta$
$1 \to 1+2$	$F_0(2\gamma/3)^2\cos^2\theta + F_1(1/9)\sin^2\theta$
$1 \to 2+1$	$2F_0\gamma^4(\cos^2\theta - 1/3)^2$
	$+F_1\gamma^2[2/9+2/3\cos^2\theta-2(\cos^2\theta - 1/3)^2]$
$1 \to 2+2$	$F_1\gamma^2\cos^2\theta\sin^2\theta$
$2 \to 1+1$	$F_0(2\gamma^2/3)\cos^2\theta^2 + F_1(1/2)\sin^2\theta$
$2 \to 1+2$	$F_1\sin^2\theta$
$2 \to 2+0$	$F_0(4\gamma^4/3 + 4\gamma^2/3 + 1/3)(\cos^2\theta-1/3)^2$
	$+F_1\gamma^2[4/9 + 4/3\cos^2\theta - 4(\cos^2\theta-1/3)^2]$
	$+F_2[8/9 - 4/3\cos^2\theta + (\cos^2\theta-1/3)^2]$
$2 \to 2+1$	$F_1\gamma^2[1/9+1/3\cos^2\theta - (\cos^2\theta-1/3)^2]$
	$+F_2[8/9 - 4/3\cos^2\theta + (\cos^2\theta-1/3)^2]$
$2 \to 2+2$	$3F_0(4\gamma^2/9-1/9)^2(\cos^2\theta-1/3)^2$
	$+F_1\gamma^2[1/9+4/3-(\cos^2\theta-1/3)^2]/9$
	$+F_2[8/9-4/3\cos^2\theta + (\cos^2\theta-1/3)^2]/9$

Table 4.3: Angular distributions for $J \neq 0, L \neq 0, l$ for *polarized* initial states where $\cos\theta$ is the angle between particles a and c in the rest frame of resonance r, $\gamma = E_r/m_{ab}$, and $E_r = (m_R^2 + m_{ab}^2 - m_c^2)/2m_R$. F_λ denotes the fraction of the initial state in helicity state λ. For unpolarized initial states setting F_λ=1 recovers the angular distributions obtained from the Zemach formalism shown in 4.2.

where I is the identity operator, $\hat{T}$ is Lorentz invariant transition operator, ρ is the diagonal phase space matrix where $\rho_{ii} = 2q_i/m$ and q_i is the momentum of a in the r rest frame for decay channel i. In the single channel S-wave scenario $S = e^{2i\delta}$ satisfies unitarity and implies

$$\hat{T} = \frac{1}{\rho} e^{i\delta} \sin\delta. \tag{4.4}$$

There are three common formulations of the dynamical function. The Breit-Wigner formalism is the simplest formulation - the first term in a Taylor expansion about a T matrix pole. The K-matrix formalism[53] is more general (allowing more than one T matrix pole and coupled channels while preserving unitarity). The Flatté distribution[66] is used to parameterize resonances near threshold and is equivalent to a one-pole, two-channel K-matrix.

4.2.4 Breit-Wigner Formulation

The common formulation of a Breit-Wigner resonance decaying to spin-0 particles a and b is

$$T_r(m_{ab}) = \frac{1}{m_r^2 - m_{ab}^2 - i m_r \Gamma_{ab}(q)} \tag{4.5}$$

where the "mass dependent" width Γ is

$$\Gamma = \Gamma_r \left(\frac{q}{q_r}\right)^{2L+1} \left(\frac{m_r}{m_{ab}}\right) B_L'(q, q_0)^2 \tag{4.6}$$

where $B_L'(q, q_0)$ is the Blatt-Weisskopf barrier factor from 4.1. A Breit-Wigner parametrization best describes isolated, non-overlapping resonances far from the threshold of additional decay channels. For the ρ and $\rho(1450)$ a more complex parametrization as suggested by Gounaris-Sakarai[67] is often used [23][28][30][35].

Unitarity can be violated when the dynamical function is parameterized as the sum of two or more overlapping Breit-Wigners. The proximity of a threshold to the resonance shape distorts the line shape from a simple Breit-Wigner. This scenario is described by the Flatté formula and is discussed below.

4.2.5 K-matrix Formalism

The T matrix can be described as

$$\hat{T} = (I - i\hat{K}\rho)^{-1}\hat{K}, \tag{4.7}$$

where $\hat{K}$ is the Lorentz invariant K-matrix describing the scattering process and ρ is the phase space factor.

Resonances appear as a sum of poles in the K-matrix

$$\hat{K}_{ij} = \sum_\alpha \frac{\sqrt{m_\alpha \Gamma_{\alpha i}(m) m_\alpha \Gamma_{\alpha j}(m)}}{(m_\alpha^2 - m^2)\sqrt{\rho_i \rho_j}}. \tag{4.8}$$

The K-matrix is real by construction thus the associated T-matrix respects unitarity.

For the special case of a single channel, single pole we obtain

$$K = \frac{m_0 \Gamma(m)}{m_0^2 - m^2} \tag{4.9}$$

and

$$T = K(1 - iK)^{-1} = \frac{m_0 \Gamma(m)}{m_0^2 - m^2 - i m_0 \Gamma(m)} \tag{4.10}$$

which is the relativistic Breit-Wigner formula. For the special case of a single channel, two poles we have

$$K = \frac{m_\alpha \Gamma_\alpha(m)}{m_\alpha^2 - m^2} + \frac{m_\beta \Gamma_\beta(m)}{m_\beta^2 - m^2} \tag{4.11}$$

and in the limit that m_α and m_β are far apart relative to the widths we can approximate the T matrix as the sum of two Breit-Wigners, $T(K_\alpha + K_\beta) \approx T(K_\alpha) + T(K_\beta)$,

$$T \approx \frac{m_\alpha \Gamma_\alpha(m)}{m_\alpha^2 - m^2 - i m_\alpha \Gamma_\alpha(m)} + \frac{m_\beta \Gamma_\beta(m)}{m_\beta^2 - m^2 - i m_\beta \Gamma_\beta(m)}. \tag{4.12}$$

In the case of two nearby resonances 4.12 is not valid and exceeds unity (and hence T violates unitarity).

This formulation, which applies to S-channel production in two-body scattering $ab \rightarrow cd$, can be generalized to describe the production of resonances in processes, such as the decay of charm mesons. The key assumption here is that the two-body system described by the K-matrix does *not* interact with the rest of the final state[56]. The quality of this assumption varies with the production process and is appropriate for scattering experiments like $\pi^- p \rightarrow \pi^0 \pi^0 n$, radiative decays such as ϕ, $J/\psi \rightarrow \gamma\pi\pi$ and semileptonic decays such as $D \rightarrow K\pi\ell\nu$. This assumption may be of limited validity for production processes such as $p\overline{p} \rightarrow \pi\pi\pi$ or $D \rightarrow \pi\pi\pi$. In these scenarios the two-body Lorentz invariant amplitude, $\hat{F}$, is given as

$$\hat{F}_i = (I - i\hat{K}\rho)^{-1}_{ij} \hat{P}_j = (\hat{T}\hat{K}^{-1})_{ij} \hat{P}_j \tag{4.13}$$

where P is the production vector which parameterizes the resonance production in the open channels.

For the $\pi\pi$ S-wave, a common formulation of the K-matrix[55][13][30] is

$$K_{ij}(s) = \left\{ \sum_\alpha \frac{g_i^{(\alpha)} g_j^{(\alpha)}}{m_\alpha^2 - s} + f_{ij}^{sc} \frac{1 - s_0^{sc}}{s - s_0^{sc}} \right\} \times \frac{s - s_A/2m_\pi^2}{(s - s_{A0})(1 - s_{A0})}. \tag{4.14}$$

The factor $g_i^{(\alpha)}$ is the real coupling constant of the K-matrix pole m_α to meson channel i; the parameters f_{ij}^{sc} and s_0^{sc} describe a smooth part of the K-matrix elements; the multiplicative factor $\frac{s - s_A/2m_\pi^2}{(s - s_{A0})(1 - s_{A0})}$ suppresses a false kinematical singularity near the $\pi\pi$ threshold - the Adler zero; and the number 1 has units GeV^2.

The production vector, with $i = 1$ denoting $\pi\pi$, is

$$P_j(s) = \left\{ \sum_\alpha \frac{\beta_\alpha g_j^{(\alpha)}}{m_\alpha^2 - s} + f_{1j}^{pr} \frac{1 - s_0^{pr}}{s - s_0^{pr}} \right\} \times \frac{s - s_A/2m_\pi^2}{(s - s_{A0})(1 - s_{A0})}, \tag{4.15}$$

where the free parameters of the Dalitz plot fit are the complex production couplings β_α, and the production vector background parameters f_{1j}^{pr} and s_0^{pr}. All other parameters are fixed by scattering experiments. Ref. [54] describes the $\pi\pi$ scattering data with a 4 pole, 2 channel ($\pi\pi$, KK) model while Ref. [55] describes the scattering data with 5 pole, 5 channel ($\pi\pi$, KK, $\eta\eta$, $\eta'\eta'$ and 4π) model. The former has been implemented by CLEO[6] and the latter by FOCUS[13] and BaBar[30]. In both cases only the $\pi\pi$ channel was analyzed. A more complete coupled channel analysis would simultaneously fit all final states accessible by rescattering.

4.2.6 Flatté Formalism

The scenario where another channel opens close to the resonance position can be described by the Flatté formulation

$$\hat{T}(m_{ab})=\frac{1}{m_r^2 - m_{ab}^2 - i(\rho_1 g_1^2 + \rho_2 g_2^2)}, \quad g_1^2 + g_2^2 = m_r \Gamma_r. \tag{4.16}$$

This situation occurs in the $\pi\pi$ S-wave where the $f_0(980)$ is near the $K\overline{K}$ threshold and in the $\pi\eta$ channel where the $a_0(980)$ also lies near $K\overline{K}$ threshold. For the $a_0(980)$ resonance the relevant coupling constants are $g_1 = g_{\pi\eta}$ and $g_2 = g_{KK}$ and the phase space terms are $\rho_1 = \rho_{\pi\eta}$ and $\rho_2 = \rho_{KK}$ where

$$\rho_{ab} = \sqrt{\left(1 - (\frac{m_a - m_b}{m_{ab}})^2\right)\left(1 + (\frac{m_a - m_b}{m_{ab}})^2\right)} \tag{4.17}$$

For the $f_0(980)$ the relevant coupling constants are $g_1 = g_{\pi\pi}$ and $g_2 = g_{KK}$ and the phase space terms are $\rho_1 = \rho_{\pi\pi}$ and $\rho_2 = \rho_{KK}$. The charged and neutral K channels are usually assumed to have the same coupling constant but separate phase space factors due to $m_{K^+} \neq m_{K^0}$ resulting in

$$\rho_{KK}=\frac{1}{2}\left(\sqrt{1-\left(\frac{2m_{K^\pm}}{m_{KK}}\right)^2}+\sqrt{1-\left(\frac{2m_{K^0}}{m_{KK}}\right)^2}\right). \tag{4.18}$$

4.2.7 Branching Ratios from Dalitz Fits

The fit to the Dalitz plot distribution using either the Breit-Wigner or the K-matrix formalism factorizes into a resonant contribution to the amplitude $\mathcal{M}_j$ and a complex coefficient, $a_j e^{i\delta_j}$, where a_j and δ_j are real. The definition of a rate of a single process, given a set of amplitudes a_j and phases δ_j is the square of the relevant matrix element (see 4.1). In this spirit, the fit fraction is usually defined as the integral over the Dalitz plot (m_{ab} vs m_{bc}) of a single amplitude squared divided by the integral over the Dalitz plot of the square of the coherent sum of all amplitudes,

$$\text{Fit Fraction}_j = \frac{\int \left|a_j e^{i\delta_j}\mathcal{M}_j\right|^2 dm_{ab}^2 dm_{bc}^2}{\int \left|\sum_k a_k e^{i\delta_k}\mathcal{M}_k\right|^2 dm_{ab}^2 dm_{bc}^2} \tag{4.19}$$

where $\mathcal{M}_j$ is defined by 4.2 and described in Ref.[4]. The sum of the fit fractions for all components will in general not be unity due to interference.

It should be noted that, when the K-matrix description in 4.13 is used to describe a wave (e.g. $\pi\pi$ S-wave) then $\mathcal{M}_j$ refers to the entire wave. In these circumstances, it may not be straightforward to separate it into a sum of individual resonances unless these are narrow and well separated, in which case 4.12 can be used.

Reconstruction Efficiency

The efficiency for reconstructing an event as a function of position on the Dalitz plot is in general non-uniform.

Describe the efficiency parametrizations used at CLEO

Typically, a signal Monte Carlo sample generated with a uniform distribution in phase space is used to determine the efficiency. The variation in efficiency across the Dalitz plot varies with experiment and decay mode. Most recent analyses utilize a full GEANT[68] detector simulation.

Finite detector resolution can usually be safely neglected as most resonances are comparatively broad. Notable exceptions where detector resolution effects must be modeled are $\phi \to K^+K^-$, $\omega \to \pi^+\pi^-$, and $a_0 \to \eta\pi^0$. One approach is to convolve the resolution function in the Dalitz-plot variables m^2_{ab}, m^2_{bc} with the function that parameterizes the resonant amplitudes. In high statistics data samples resolution effects near the phase space boundary typically contribute to a poor goodness of fit. The momenta of a,b and c *can* be recalculated with a R mass constraint. This forces the kinematical boundaries of the Dalitz plot to be strictly respected. If the three-body mass is not constrained, then the efficiency (and the parametrization of background) may also depend on the reconstructed mass. In fits to multi-body decays of charmonia and bottomonia it is not appropriate to constrain the mass due to the finite natural width of the parent.

Background Parametrization

The contribution of background to the charm and B samples varies by experiment and final state. The background naturally falls into five categories: (i) purely combinatoric background containing no resonances, (ii) combinatoric background containing intermediate resonances, such as a real K^{*-} or ρ, plus additional random particles, (iii) final states containing identical particles as in $D^0 \to K^0_S\pi^0$ background to $D^0 \to \pi^+\pi^-\pi^0$ and $B \to D\pi$ background to $B \to K\pi\pi$, (iv) mistagged decays such as a real $\overline{D}^0$ or $\overline{B}^0$ incorrectly identified as D^0 or B^0 and (v) particle misidentification of the decay products such as $D^+ \to \pi^-\pi^+\pi^+$ or $D^+_s \to K^-K^+\pi^+$ reconstructed as $D^+ \to K^-\pi^+\pi^+$.

The contribution from combinatoric background with intermediate resonances is distinct from the resonances in the signal because the former do *not* interfere with the latter since they are not from true resonances. Additionally, processes such as $\psi' \to \gamma\chi_{c2} \to \gamma(\gamma J/\psi) \to \gamma\gamma(\pi\pi)$ and $\psi' \to \pi^0 J/\psi, J/\psi \to \pi\pi$, do *not* interfere since electromagnetic and hadronic transitions proceed on different time scales. Similarly, $D^0 \to \rho\pi$ and $D^0 \to K^0_S\pi^0$ do not interfere since strong and weak transitions proceed on different time scales. The usual identification tag of the initial particle as a D^0 or a $\overline{D}^0$ is the charge of the distinctive slow pion in the decay sequence $D^{*+} \to D^0\pi_s^+$ or $D^{*-} \to \overline{D}^0\pi_s^-$. Another possibility is the identification or "tagging" of one of the D mesons from $\psi(3770) \to D^0\overline{D}^0$ as is done for B mesons from $\Upsilon(4S)$. The mistagged background is subtle and may be mistakenly enumerated in the *signal* fraction determined by a D^0 mass fit. Mistagged decays contain true $\overline{D}^0$'s or $\overline{B}^0$'s and so the resonances in the mistagged sample exhibit interference on the Dalitz plot.

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