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A new determination of the QED coupling $\alpha(M_Z^2)$ lets the Higgs off the hook

A.D. Martin[a], J. Outhwaite[a] and M.G. Ryskin[a,b]

[a] Department of Physics, University of Durham, Durham, DH1 3LE, UK.
[b] Petersburg Nuclear Physics Institute, 188350, Gatchina, St. Petersburg, Russia.

Abstract

We use the available data on $e^+e^- \to$ hadrons to determine the value of the QED coupling at the Z pole. The inclusion of recent preliminary BES-II data in the analysis is found to reduce the ambiguity in the determination of the coupling, particularly that arising from the use of either exclusive or inclusive data in the energy region $\sqrt{s} \lesssim 2$ GeV, in favour of a larger value of $\alpha(M_Z^2)^{-1} = 128.978 \pm 0.027$. As a consequence the predicted value of the mass of the (Standard Model) Higgs boson is increased, so that the preferred value is just above the LEP2 bound.

The value of the QED coupling at the Z pole, $\alpha(M_Z^2)$, is the poorest known of the three parameters $(G_F, M_Z, \alpha(M_Z^2))$ which define the standard electroweak model. Indeed it is the precision to which we know $\alpha(M_Z^2)$ which limits the accuracy of the indirect prediction of the mass M_H of the (Standard Model) Higgs boson [1]. Indeed the predicted allowed domain of M_H is not far from being in conflict with the direct LEP2 bound on the mass, so a more precise determination of $\alpha(M_Z^2)$ is especially important.

The value of $\alpha(M_Z^2)$ is obtained from

$$\alpha^{-1} \;\equiv\; \alpha(0)^{-1} \;=\; 137.03599976(50) \tag{1}$$

using the relation

$$\alpha(s)^{-1} \;=\; \left(1 - \Delta\alpha_{\rm lep}(s) - \Delta\alpha_{\rm had}^{(5)}(s) - \Delta\alpha^{\rm top}(s)\right)\; \alpha^{-1}, \tag{2}$$

where the leptonic contribution to the running of the α is known to 3 loops [2]

$$\Delta\alpha_{\rm lep}(M_Z^2) \;=\; 314.98 \times 10^{-4}. \tag{3}$$

From now on we omit the superscript (5) on $\Delta\alpha_{\rm had}$ and assume that it corresponds to five flavours. We will include the contribution of the sixth flavour, $\Delta\alpha^{\rm top}(M_Z^2) = -0.76 \times 10^{-4}$, at the end. To determine the hadronic contribution we need to evaluate

$$\Delta\alpha_{\rm had}(s) \;=\; -\frac{\alpha s}{3\pi}\, P \int_{4m_\pi^2}^{\infty} \frac{R(s')ds'}{s'(s'-s)} \tag{4}$$

at $s = M_Z^2$, where $R = \sigma(e^+e^- \to {\rm hadrons})/\sigma(e^+e^- \to \mu^+\mu^-)$.

The early determinations of $\Delta\alpha_{\rm had}(M_Z^2)$ made maximum use of the e^+e^- measurements of $R(s)$, using the sum of the exclusive hadronic channels ($e^+e^- \to 2\pi, 3\pi, \ldots K\bar{K}, \ldots$) for $\sqrt{s} \lesssim 1.5$ GeV and the inclusive measurement of $\sigma(e^+e^- \to {\rm hadrons})$ at larger energies, culminating in the analysis of Eidelman and Jegerlehner [3] which gave

$$\Delta\alpha_{\rm had}(M_Z^2) \;=\; (280.4 \pm 6.4) \;\times\; 10^{-4}. \tag{5}$$

Other similar determinations, which gave compatible results, can be found in Refs. [4, 5, 6], where the latter analysis incorporated information from $\tau \to ({\rm hadrons} + \nu_\tau)$ decays. However, in the last few years the determinations of $\Delta\alpha_{\rm had}$ from (4) have relied more and more on theoretical input. First, perturbative QCD was used to better describe $R(s)$ (for $\sqrt{s} > 3$ GeV) in continuum energy regions above the resonances up to the next flavour threshold [7]. Then, encouraged by the success of perturbative QCD to describe τ decay, it was used down to 1.8 GeV, across a region with sparse data on $R(s)$, giving [8]

$$\Delta\alpha_{\rm had}(M_Z^2) \;=\; (277.5 \pm 1.7) \times 10^{-4}, \tag{6}$$

where $\pm 10^{-4}$ comes from the uncertainty in perturbative QCD. A recent analysis [9], which follows the more conservative perturbative QCD input of [7], finds

$$\Delta\alpha_{\rm had}(M_Z^2) \;=\; (274.18 \pm 2.52) \;\times\; 10^{-4} \quad \text{(inclusive)} \tag{7}$$

$$\Delta\alpha_{\rm had}(M_Z^2) \;=\; (276.97 \pm 2.90) \;\times\; 10^{-4} \quad \text{(exclusive)} \tag{8}$$

according to whether R is evaluated in the interval $1.46 < \sqrt{s} < 2.125$ GeV using either the inclusive data for $e^+e^- \to$ hadrons or the sum of the data for the exclusive channels. The difference in the input in this region can be seen in Fig. 1.

Other recent calculations of $\alpha(M_Z^2)$, based on (4), have used the analytic behaviour in the complex s plane in attempts to reduce the error coming from the measurements of R, see, for example, Refs. [10]–[12]. These methods have been critically reviewed by Jegerlehner [13]. In this work he showed that a space-like evaluation of (4) at $s = -(2.5\ {\rm GeV})^2$, followed by an analytic continuation [14] to $s = -M_Z^2$, and then around the semicircle to $s = M_Z^2$, gives

$$\Delta\alpha_{\rm had}(M_Z^2) \;=\; (277.82 \pm 2.54) \;\times\; 10^{-4}, \tag{9}$$

in agreement with the direct determination of (4) at $s = M_Z^2$. He concludes that no appreciable reduction of the error, arising from the data, is obtained by the analytic methods. The reduction in the error in (7–9), in comparison to that in (5), is due to the use of perturbative QCD to evaluate R in energy regions where it is expected to be reliable.

We see that the values (6), (8) and (9) are in good agreement with each other, but that the determination (7) using inclusive data below 2.1 GeV is significantly lower. This discrepancy has existed for some time, although most recent determinations have relied on the sum of the exclusive data. However BES-II 1999 data have recently been presented [15] which, although preliminary, already clarify the behaviour of $R(s)$ in the critical $2 \lesssim \sqrt{s} \lesssim 3$ GeV domain. These data update the previous, published, BES-II measurements [16]. They are shown by the solid circular data points in Fig. 1. First, we see that in the region $\sqrt{s} \sim 2.5$ GeV the new measurements are closer to the expectations of perturbative QCD than the previous measurements. Second, at $\sqrt{s} = 2 - 2.2$ GeV they join reasonably smoothly with the inclusive measurements of the $\gamma\gamma 2$ collaboration [17], whereas they lie in the region of, or below, the lower limit of the determination of R from the sum of the exclusive channels.

Here we re-evaluate (4) at $s = M_Z^2$, incorporating these new BES-II data[1]. To be specific we use the dashed curve for $\sqrt{s} > 1.46$ GeV for the so-called "inclusive" determination of $\Delta\alpha_{\rm had}(M_Z^2)$, and the average of the bounds for $\sqrt{s} < 1.9$ GeV for the "exclusive" determination. In addition we use the data, and follow the method, described in [7, 9]. The errors on the input 'data' values of $R(s')$ are calculated using a correlated χ^2 minimization to combine different data sets, as described in detail in Ref. [6].

The data from $\sqrt{s} = 2.8$ GeV up to the $\mathrm{D}\overline{\mathrm{D}}$ charm meson threshold are in good agreement[2] with perturbative QCD, apart, of course, in J/ψ and ψ' resonance regions. In the energy intervals where perturbative QCD is believed to be valid ($2.8 < \sqrt{s} < 3.74$ GeV and $\sqrt{s} >$ 5 GeV) we use both the two-loop expression with the quark mass explicity included and the massless three-loop expression [20] calculated in the $\overline{\mathrm{MS}}$ renormalization scheme[3]. In addition to the above, we include the perturbative QCD error[4] coming from varying m_c, m_b, M_Z within the uncertainties quoted in the [22], $\alpha_S(M_Z^2) = 0.119 \pm 0.002$ and varying the scale $\alpha_S(cs)$ in the range $0.25 < c < 4$. The contributions to $\Delta\alpha_{\rm had}(M_Z^2)$ of (4) from the various s' intervals are listed in Table 1.

The recent Novosibirsk measurements have considerably improved the exclusive data below 1.4 GeV, see, for example, Refs. [23, 24]. For instance, the measurements of $e^+e^- \to \pi^+\pi^-$ [23] are of much improved precision such that, when taken together with chiral perturbation theory estimates, we find that the error on the 2π contribution to $\Delta\alpha_{\rm had}$ for $\sqrt{s} < 0.96$ GeV is reduced to about $\pm 0.5 \times 10^{-4}$. We checked that our results for the contributions from the exclusive channels were in agreement with the detailed table of results given in Ref. [6], if we were to omit the new Novosibirsk data.

From Table 1 we see that the largest uncertainties in $\Delta\alpha_{\rm had}$ occur in the $1.46 < \sqrt{s'} <$ 2.8 GeV interval, although the introduction of the new BES-II data [15] have led to a significant improvement. It is interesting to note that if we were to assume that perturbative QCD gave

[1] We note a recent preliminary value of $\Delta\alpha_{\rm had}(M_Z^2) = (275.5 \pm 4.6) \times 10^{-4}$ presented at ICHEP 2000 by B. Pietrzyk [18], which also incorporates the new BES-II data, but in comparison with the present analysis, uses purely the experimental measurements through the low energy region.

[2] The MARK I data [19] are normalized as described in [7].

[3] The uncertainty due to using a different scheme may be estimated to be of the order of the $\mathcal{O}(\alpha_S^4)$ correction, which is about $3\sum e_q^2 r_3(\alpha_S/\pi)^4$. We may take $r_3 = -128$ [21] which leads to an uncertainty much smaller than that given in Table 1.

[4] We thank Thomas Teubner for valuable discussions concerning the perturbative QCD contribution.

a reliable prediction for $R(s')$ in the region $1.9 < \sqrt{s'} < 2.8$ GeV, then the contribution from this interval would be $(13.18 \pm 0.09) \times 10^{-4}$. This is essentially the same as the "inclusive" contribution, but with a much smaller error. The "exclusive" contribution is a little larger on account of the necessity to smoothly match up to the "exclusive" data at $\sqrt{s} = 1.9$ GeV.

Table 1: Contributions to $\Delta\alpha_{\rm had}(M_Z^2) \times 10^4$ of (4) coming from the different $\sqrt{s'}$ intervals. The alternative values in the round brackets use the summation of exclusive channels as the contribution from the region 1.46–1.9 GeV, rather than that from the inclusive data for $R(s')$. Perturbative QCD is used to evaluate the contributions in the intervals $2.8 < \sqrt{s'} < 3.74$ and $\sqrt{s'} > 5$ GeV. The errors on these contributions are described in the text.

$\sqrt{s'}$ interval (GeV)	Contribution to $\Delta\alpha_{\rm had} \times 10^4$
$2m_\pi - 1.46^a$	$38.41 \pm \left\{ \begin{array}{c} 0.52 \\ 0.60^b \end{array} \right\}$
1.46 - 1.9	8.66 ± 0.60^c
	$\left(10.32 \pm 1.06^b\right)$
1.9 - 2.8	13.24 ± 0.88^c
	(13.88 ± 0.88)
2.8 - 3.74	9.73 ± 0.05^d
3.74 - 5	15.02 ± 0.49
5 - ∞	169.97 ± 0.64^d
ω, ϕ, ψ's, Υ's	18.79 ± 0.58
$\Delta\alpha_{\rm had}^{(5)} \times 10^4$	273.82 ± 1.97
	(276.12 ± 2.20)
$\alpha^{-1}(M_Z^2)$	128.978 ± 0.027
	(128.946 ± 0.030)

[a] The upper (lower) error corresponds to the 2π (remaining) exclusive channels.
[b,c,d] Errors with identical superscripts are added linearly. The remaining errors are added in quadrature.
[d] For the pQCD contribution we take the mass of the charm quark $m_c = 1.46$ GeV, and the scale μ^2 of the QCD coupling α_S to be s'.

To conclude, we see that the introduction of new data has improved the error of $\Delta\alpha_{\rm had}$, and moved the "inclusive" and "exclusive" determinations a little closer together (compare (7) and (8) with the values of $\Delta\alpha_{\rm had}$ in Table 1). Second, the new BES-II data appear to join more smoothly to the inclusive data than the exclusive measurements. However the selection is not conclusive and there remains a major residual uncertainty in the $1.5 < \sqrt{s} < 1.9$ GeV interval, which is reflected by the two different results for $\Delta\alpha_{\rm had}$ listed in Table 1. However if we take the favoured "inclusive" result

$$\Delta\alpha_{\rm had}^{(5)} \;=\; (273.82 \pm 1.97) \;\times\; 10^{-4}, \tag{10}$$

then

$$\alpha^{-1}(M_Z^2) \;=\; 128.978 \pm 0.027. \tag{11}$$

Using this value, the latest χ^2 profile, obtained[5] by the LEP and SLD Electroweak Working

[5] We thank Martin Grünewald for making this plot.

Group [18, 25], for different values of the mass of the (Standard Model) Higgs boson, is shown by the dashed line in Fig. 2. We see that it produces a significant increase in the predicted mass of the Higgs, with the preferred value lying just above the LEP2 excluded region. The new χ^2 profile accommodates the LEP2 bound on the mass more comfortably. Note that even the determination using the exclusive data up to $\sqrt{s} = 1.9$ GeV, now predicts a *lower* value of $\Delta\alpha_{\rm had}$,

$$\Delta\alpha^{(5)}_{\rm had} \;=\; (276.12 \pm 2.20) \;\times\; 10^{-4}, \tag{12}$$

and a *higher* Higgs mass than before, with the corresponding χ^2 profile curve being nearer to the dashed curve than the original continuous curve in Fig. 2.

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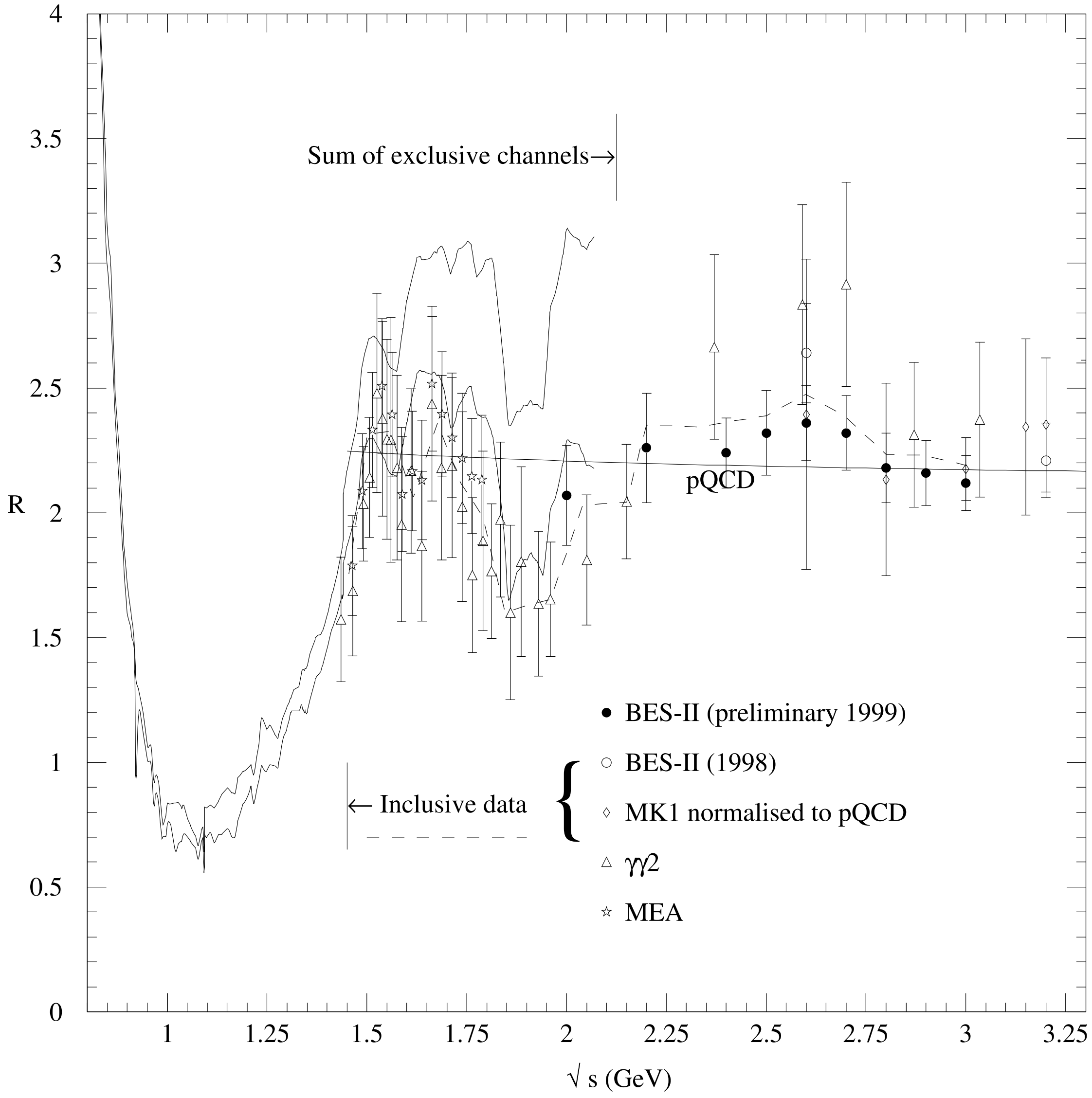

Figure 1: A plot of the ratio $R(s)$ versus $\sqrt{s}$ in the most sensitive $\sqrt{s}$ interval. Up to $\sqrt{s} = 2.125$ GeV we show by continuous lines the upper and lower bounds of the sum of the exclusive channels. However in the "exclusive" analysis we only use these data up to $\sqrt{s} = 1.9$ GeV. The value quoted in Table 1 corresponds to the average of the bounds. Above $\sqrt{s} = 1.46$ GeV we show the inclusive measurements of $R(s)$, together with the interpolation (dashed curve) used in (4). For $2.8 < \sqrt{s} < 3.74$ perturbative QCD is used to evaluate R; the central QCD prediction is shown by the continuous curve. In the region $1.46 < \sqrt{s} < 1.9$ we used, in turn, the inclusive and exclusive data to evaluate $\Delta\alpha_{\rm had}$, see Table 1.

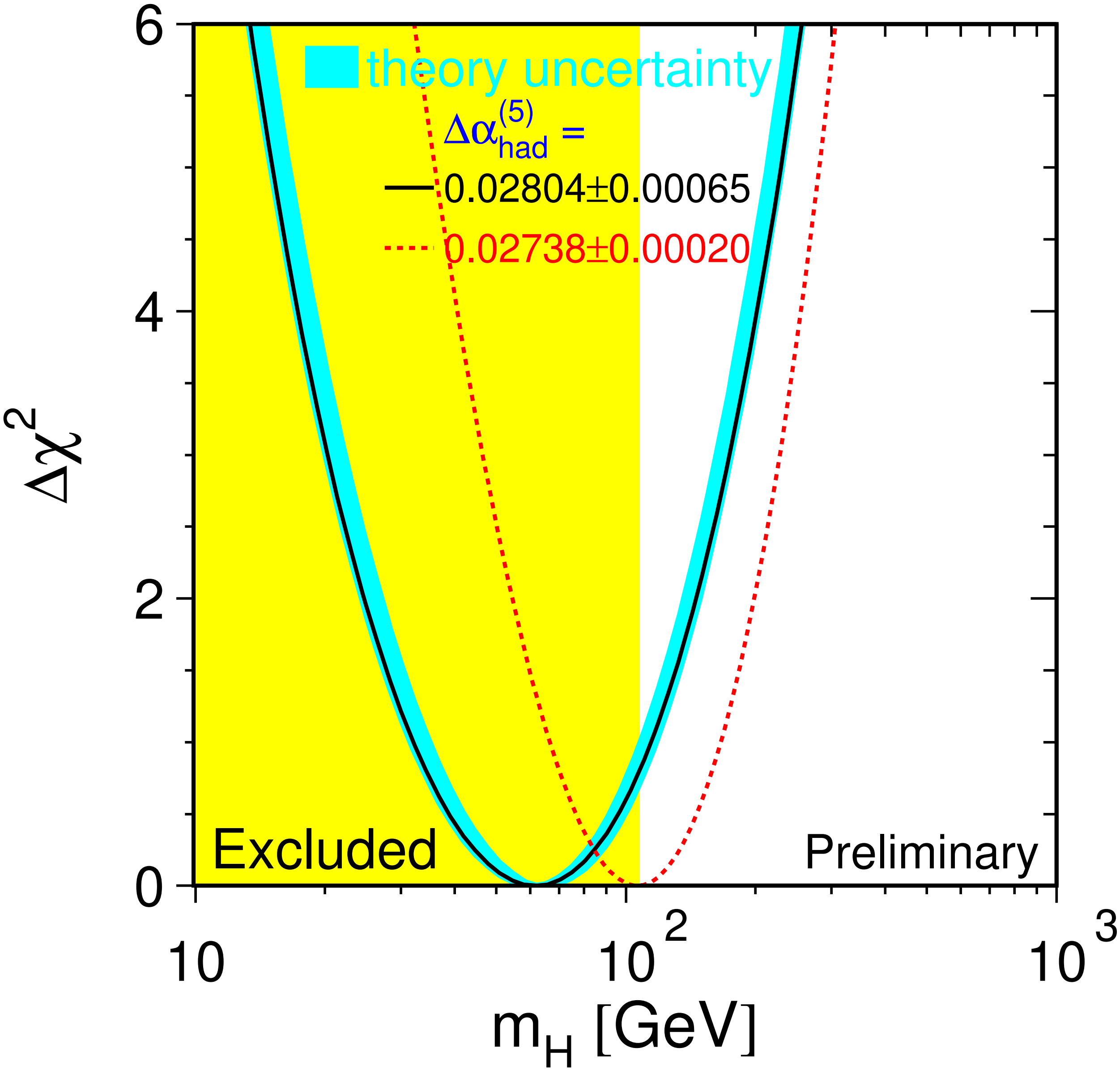

Figure 2: The latest χ^2 profile versus the Standard Model Higgs mass obtained by the LEP and SLD Electroweak Working Group [18, 25], which includes the latest preliminary experimental data, using $\Delta\alpha_{\rm had}$ of (5) compared to that obtained using (10), shown by continuous and dashed curves respectively. The shaded region to the left is excluded by searches for the Higgs boson at LEP2.