$\mathbf{a_\mu(had);\ \alpha(M_Z);\ R_{pQCD}}$
Not a Review

Kühn, U. Karlsruhe

I - $a_\mu(had)$ and $\alpha(M_Z)$
similarities and differences

II - τ vs e^+e^--data

III - R_{pQCD} below 10 GeV

IV - R_{had} from tagged photon events

V - SUMMARY

UNIVERSITÄT KARLSRUHE (TH)

$\mathrm{a}_\mu(\mathrm{had})$ and $\alpha(\mathrm{M_Z})$
similarities and differences

precision $(a_\mu \times 10^{11})$

exp. result (BNL): $\delta a_\mu = \begin{cases} 151 & \texttt{now} \\ 40 & \texttt{final} \end{cases}$

$a_\mu(\texttt{weak}) = 150$

$a_\mu(\texttt{new}) = \ ?$

$a_\mu(\texttt{exp}) - a_\mu(\texttt{th}) = 426 \pm 165$

$$a_\mu(\texttt{had}) = \begin{cases} 7022.9 \pm 58.5 \pm 140.9 & \text{[EJ, 95]}(e^+e^-) \\ 6924 \pm 62 & \text{[DH, 98]}(e^+e^-, \tau, \dots) \end{cases}$$

$$a_\mu(\texttt{had}) \sim \int \underbrace{ds \frac{K(s)}{s}}_{\sim ds/s^2} R(s)$$

dominated by low energies

70% from $\pi\pi$

90% from $\lesssim 1.8$ GeV

$$\Delta\alpha(M_Z)_{had} \sim \int \underbrace{\frac{ds}{s}}_{\sim\, d(\ln s)} \; R(s) \; \underbrace{\frac{1}{s - M_Z^2}}_{\text{flat}}$$

benchmarks:

$$\frac{\delta M_W}{M_W} \approx -\,0.20\;\delta\Delta\alpha$$

$$\frac{\delta \sin^2\theta_f}{\sin^2\theta_f} \approx \;\; 1.4\;\delta\Delta\alpha$$

$$\frac{\delta M_H}{M_H} \approx -\,\frac{\delta\Delta\alpha}{20} \times 10^4 \qquad (\sin^2\theta_{\mathtt{eff}}\;\; \text{fixed})$$

$$\Delta\alpha_{had}\downarrow \;\Rightarrow\; M_H\uparrow$$

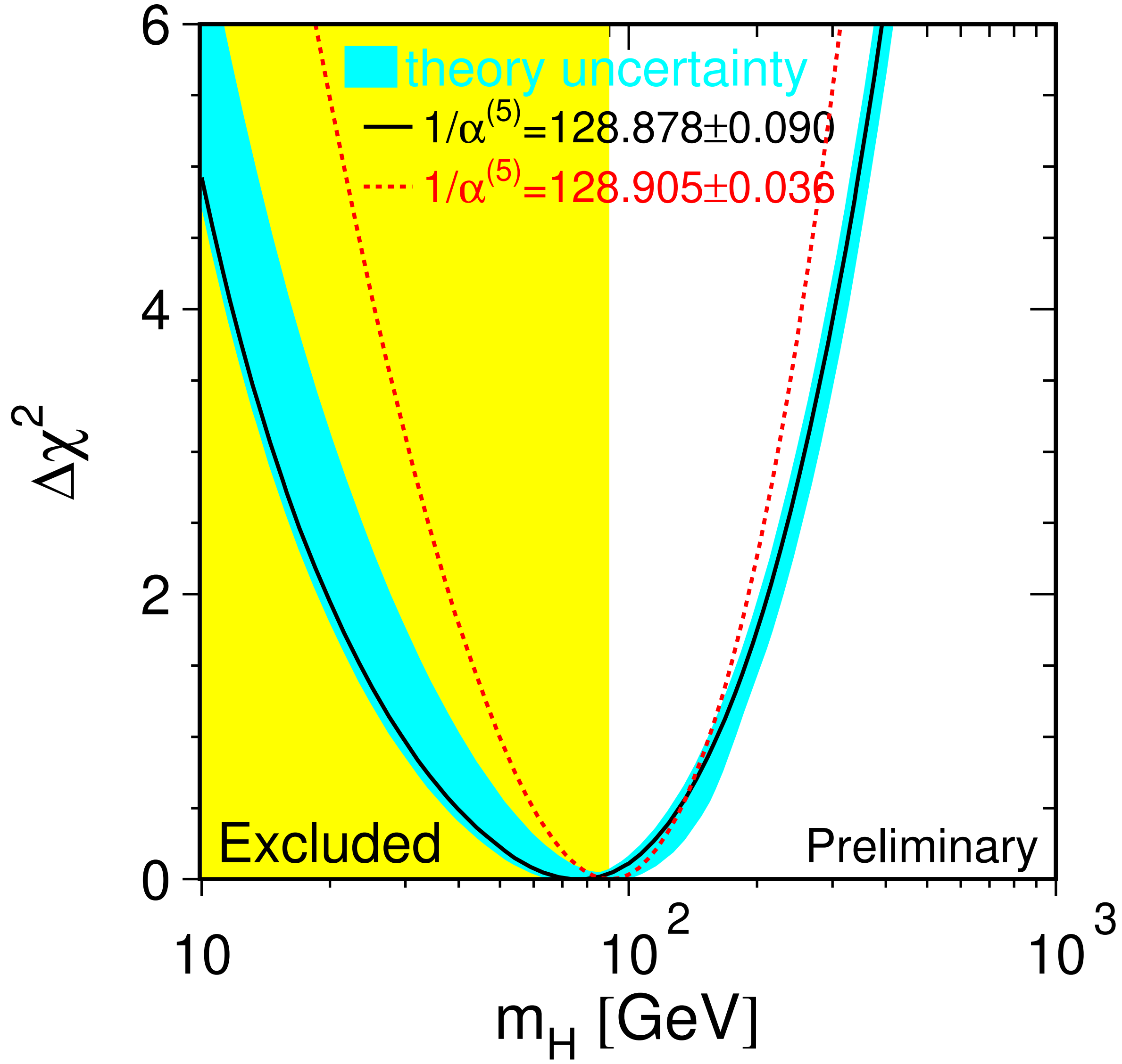
LEP-EWWG: $7 \to 2 \times 10^{-4}$

significant effect on $M_H \to$ figure

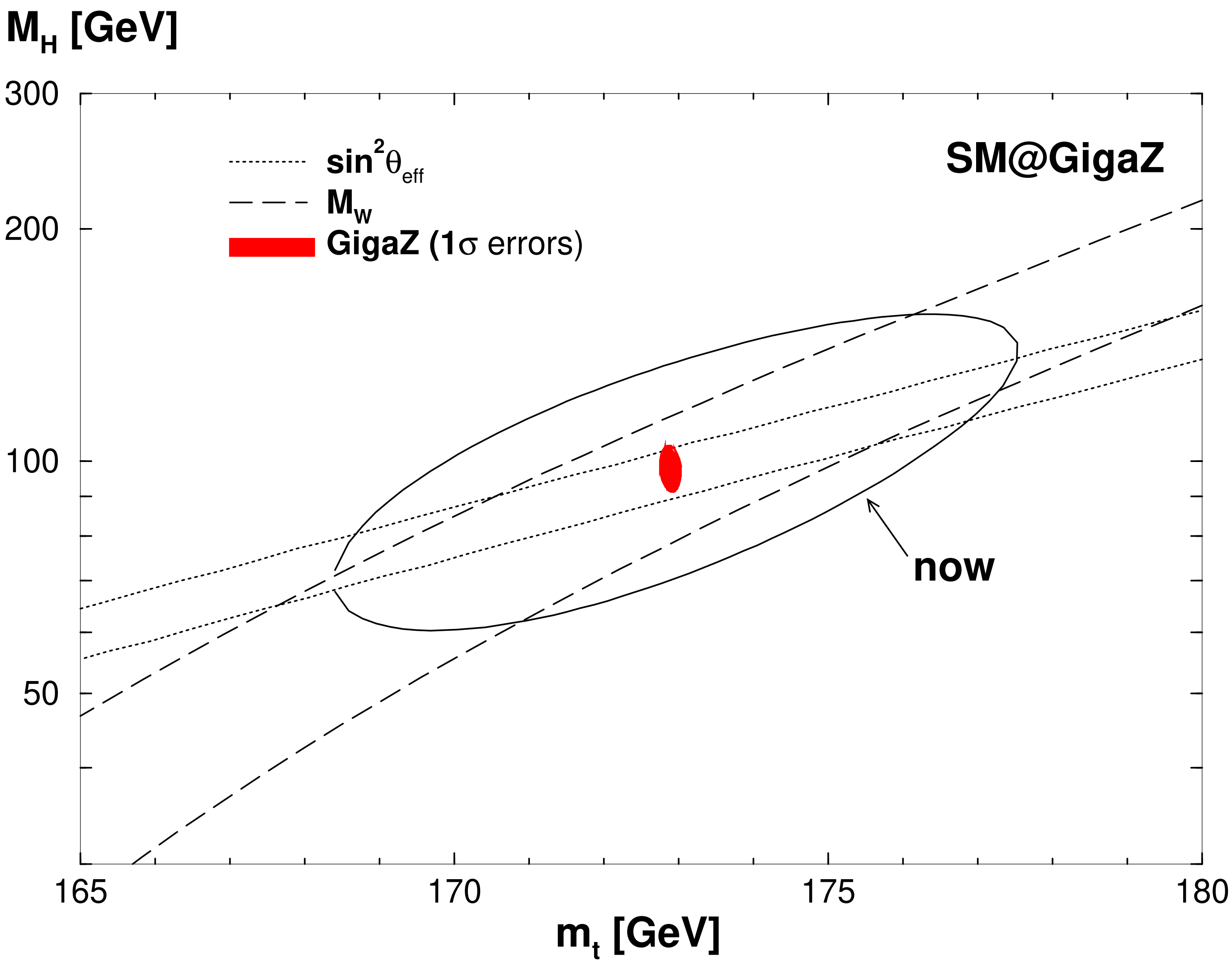
GIGA-Z: assume $0.7 \times 10^{-4} \to$ figure

some results $\to$ table

	20% below 1.8 GeV
Contributions:	70% pQCD
	10% rest



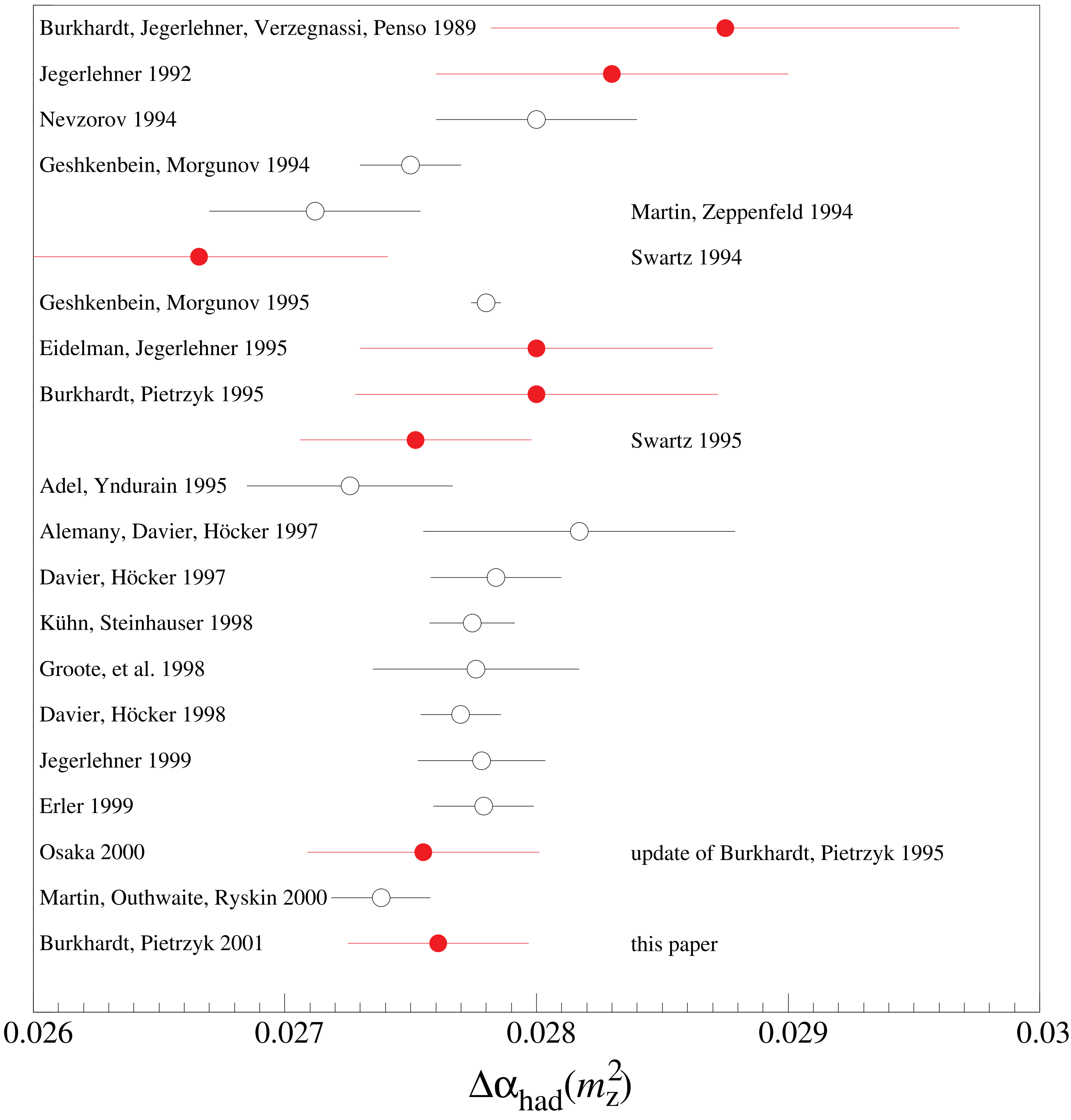
theory uncertainty
$1/\alpha^{(5)}$=128.878±0.090
$1/\alpha^{(5)}$=128.905±0.036
Excluded
Preliminary
$\Delta\chi^2$
m_H [GeV]
6
4
2
0
10
10^2
10^3



[Erler *et al.*, 00]

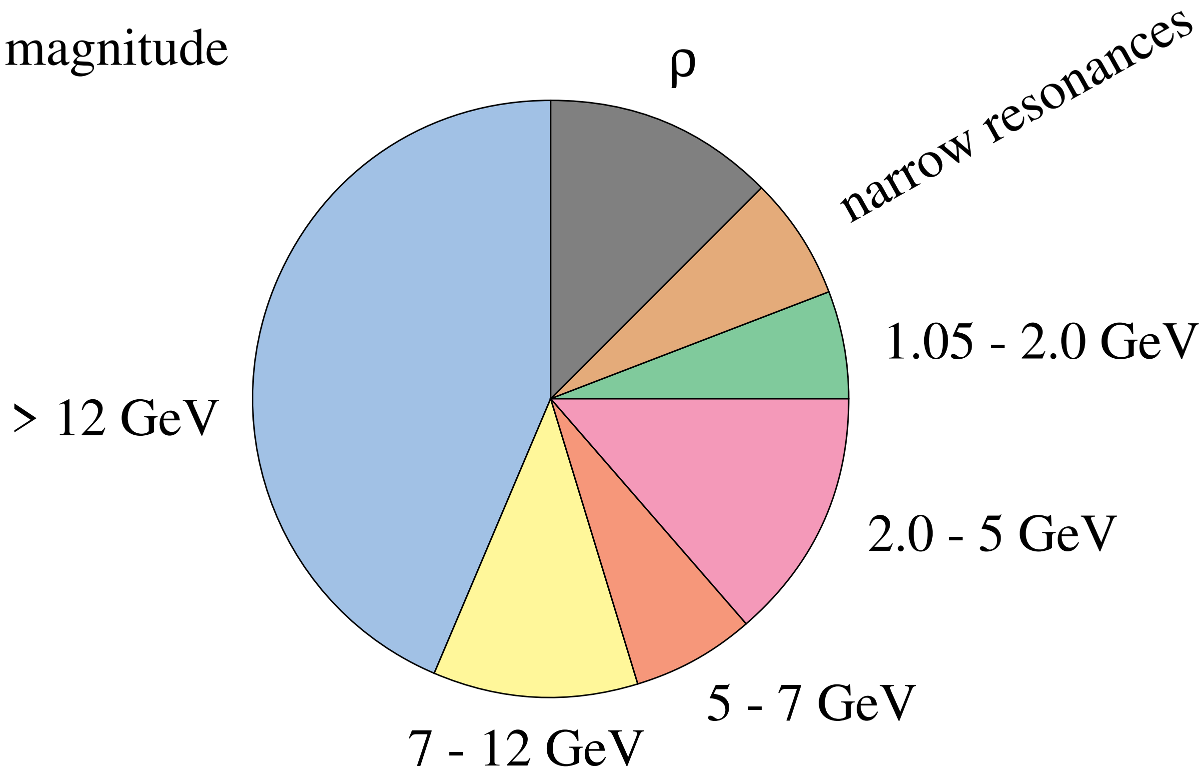
$\Delta\alpha_{\mathrm{had}}(M_Z^2)$

[Burkhardt, Pietrzyk, 01]

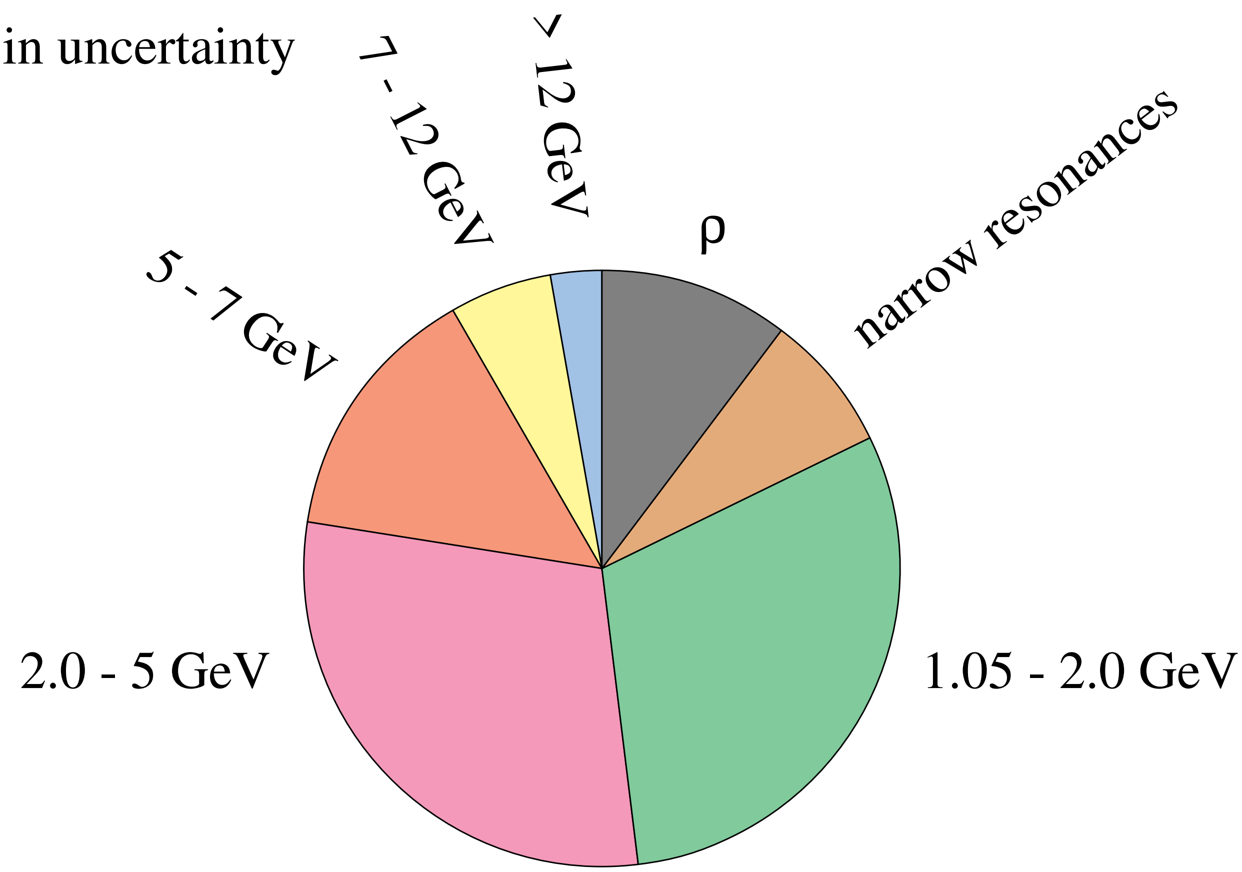


contributions at m_Z Burkhardt, Pietrzyk 2001

in magnitude



in uncertainty



$$\delta\Delta\alpha(\mathtt{had}) = \begin{cases} 7\times 10^{-4} & [\mathsf{EJ,95}](e^+e^-) \\ 3.6\times 10^{-4} & [\mathsf{BP,01}](e^+e^-) \\ 1.7\times 10^{-4} & [\mathsf{KS,98}](\tau + \mathtt{pQCD}) \end{cases}$$

poor data in regions where pQCD should be valid

[Martin, Zeppenfeld, 95]

[Chetyrkin *et al.*, 97]

[Davier, Höcker, 97]

[K, Steinhauser, 98]

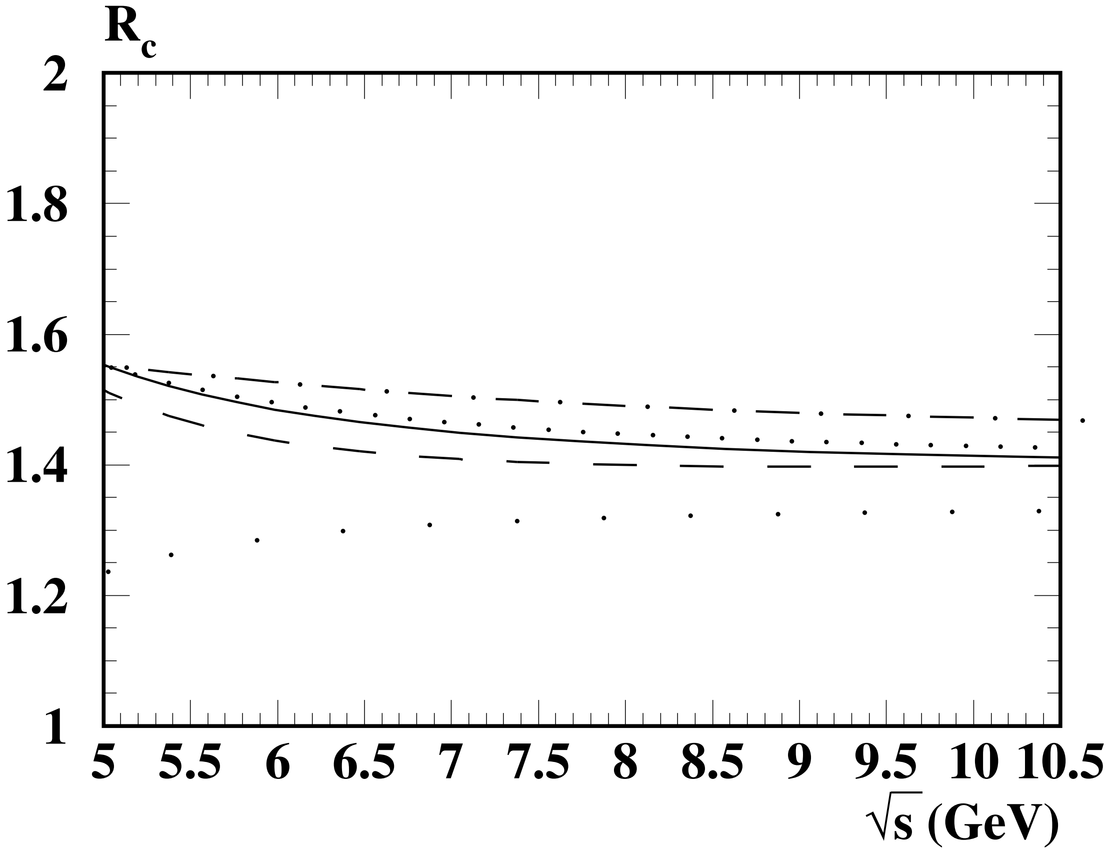
originally: $m_q = 0$

pQCD + local duality is used at the 0.5% level !

should be tested a comparable level

validity close to charm threshold?

➪ $\mathcal{O}(\alpha_s^2)$ with full m_c dependence



[CHKST, 97]

[CHKST, 97][K, Steinhauser, 98]

Energy range (GeV)	central value	$\delta\mu$	$\delta\alpha_s$	δM_c	δM_b
$1.800 - 2.125$	5.67	0.22	0.04	0.00	0.00
$2.125 - 3.000$	11.66	0.21	0.06	0.01	0.00
$3.000 - 3.700$	7.03	0.06	0.03	0.00	0.00
$1.800 - 3.700$	24.36	0.48	0.13	0.01	0.01
$5.000 - 5.500$	5.44	0.03	0.03	0.06	0.00
$5.500 - 6.000$	4.93	0.03	0.02	0.04	0.00
$6.000 - 9.460$	25.45	0.11	0.08	0.10	0.00
$9.460 - 10.520$	5.90	0.02	0.01	0.01	0.00
$10.520 - 11.200$	3.48	0.01	0.01	0.00	0.00
$5.000 - 11.200$ (without $b\bar{b}$)	45.20	0.19	0.15	0.21	0.01
$11.200 - 11.500$	1.63	0.00	0.01	0.00	0.00
$11.500 - 12.000$	2.62	0.00	0.01	0.00	0.00
$12.000 - 13.000$	4.93	0.01	0.01	0.00	0.00
$13.000 - 40.000$	72.92	0.08	0.12	0.02	0.02
$12.000 - 40.000$	77.85	0.09	0.14	0.02	0.02
$40.000 - \infty$	42.67	0.03	0.06	0.00	0.00
$11.200 - \infty$	124.77	0.12	0.21	0.03	0.02
$1.8 - \infty$ (pQCD)	194.33	0.79	0.49	0.24	0.03
QED	0.11	–	–	–	–

Contributions to $\Delta\alpha^{(5)}_{\rm had}(M_Z^2)$ (in units of 10^{-4}) from the energy regions where pQCD is used.

individual contributions:

$\langle$ 1.8 GeV - 3.7 GeV $\rangle$ = 24.36 $\pm$ 0.48 $(\delta\mu)\pm$ 0.13 $(\delta\alpha_s)$

$\langle$ 1.8 GeV - ∞, without c and b threshold $\rangle$ =

= 194.33 $\pm$ 0.79 $(\delta\mu)\pm$ 0.49 $(\delta\alpha_s)\pm$ 0.24 (δm_c)

between 2 and 3 GeV even 2% precision would be instructive!

pQCD in τ decays only globally tested

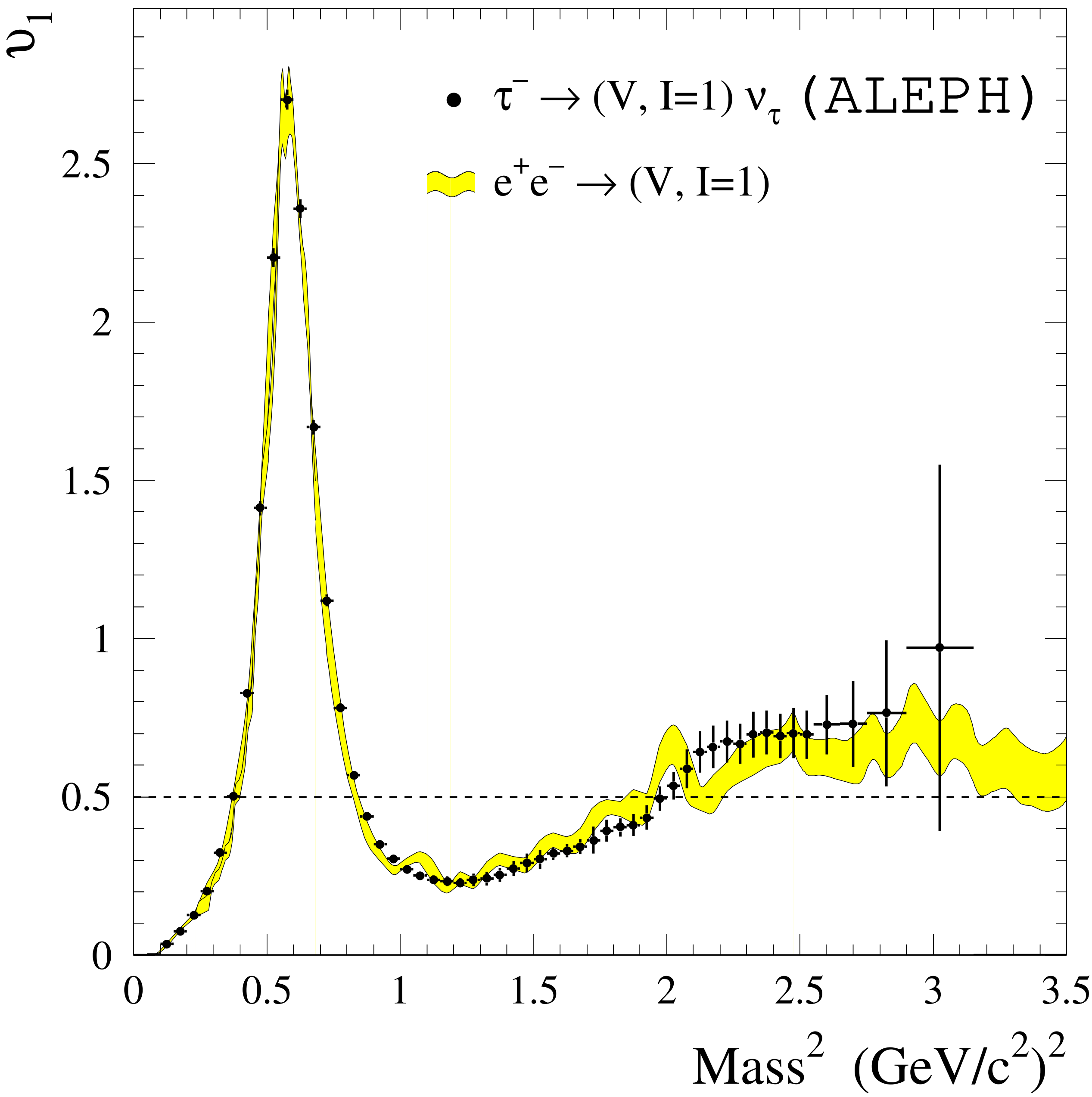
$$\int dQ^2 \frac{d\Gamma}{dQ^2}$$

$\rightarrow$ figure [Höcker]

sizeable error above 1.5 GeV

TESTING CVC (III)

Relate isovector e^+e^- cross section to τ vector spectral function:



$\mathbf{a}_\mu$(had) vs $\Delta\alpha$

formally similar   physics qualitatively different

major improvements

$\mathtt{a}_\mu(\mathtt{had})$: : e^+e^- replaced by τ

$\Delta\alpha$: : e^+e^- replaced by pQCD

(+normalization of charm - threshold)

final goal $\delta\mathrm{a}_\mu$(exp) = 40.

Minimal requirements:

$\mathtt{a}_\mu(\pi\pi) \approx$ 5000; $\delta\mathtt{a}_\mu(\pi\pi) \lesssim$ 25 ($\hat{=}0.5\%$)

now: 125 e^+e^-

60 τ

$\mathtt{a}_\mu(\mathtt{rest}) \approx$ 2000; $\delta\mathtt{a}_\mu(\mathtt{rest}) \lesssim$ 20 ($\hat{=}1\%$)

many small contributions

4π, ω, ...

improvement from PEP-N?

charm threshold

additional trick

normalization of charm threshold from 3.7 and 5 GeV (pQCD) [Kühn, Steinhauser, 98] $\longrightarrow$ table

confirmed by BES
BES: $\Delta\alpha^{(5)}_{c\bar{c}} \times 10^4 = 15.81 \pm 0.42$

or

$\Pi(-Q^2)$ with $Q^2 = (2.5\ \mathrm{GeV})^2$
spacelike argument [Jegerlehner *et al.*] ➪ m_c as input

[Martin *et al.*] find sizeable sensitivity on m_c.
Use stability of $\Pi(Q^2)$ as function of Q^2
➪ distinction between data (consistency with pQCD) and determine $m_c|_{pole} =$ 1.33-1.40 GeV

[Kühn, Steinhauser, 98]

	PLUTO	DASP	MARK1
Interval below (GeV)	$3.6000 - 3.6600$	$3.6025 - 3.6500$	$3.0000 - 3.6500$
n_-	1.04 ± 0.01	1.06 ± 0.02	1.18 ± 0.04
R_-	2.25 ± 0.03	2.29 ± 0.04	2.55 ± 0.08
Interval above (GeV)	$4.9800 - 4.9800$	$5.0000 - 5.1950$	$5.1000 - 6.0000$
n_+	1.04 ± 0.01	1.15 ± 0.01	1.12 ± 0.01
R_+	3.85 ± 0.04	4.27 ± 0.04	4.14 ± 0.05
$\Delta\alpha_{c\bar{c}}^{(5)}(M_Z^2) \times 10^4$ (Model 1)	15.65 ± 0.19	15.26 ± 0.25	16.22 ± 0.32
$\Delta\alpha_{c\bar{c}}^{(5)}(M_Z^2) \times 10^4$ (Model 2)	15.64 ± 0.16	15.68 ± 0.30	15.83 ± 0.36

Contribution to $\Delta\alpha_{\mathrm{had}}^{(5)}(M_Z^2)$ from the energy interval 3.7 to 5.0 GeV.

τ vs e^+e^- - data

originally :

[Sakurai, Thaker]

[Tsai]

[Gilman, Miller, Rhie]

[K, Santamaria]

$$e^+e^- \Rightarrow \tau$$

now:

τ as sustitute for e^+e^-

$$B(\tau \to \nu\pi\pi) = 25.31 \pm 0.18\%$$

0.6% relative precision

radiative corrections ?

isospin-breaking ?

A) QED / short distances

$$\delta_{EW} = \underbrace{1 + \frac{\alpha}{\pi} \ln \frac{M_W^2}{m_\tau^2}}_{1.019}$$

B) QED / intermediate distances

$$\sim \frac{\alpha}{\pi} \ln \frac{m_\tau^2}{M^2(?)} \Rightarrow 0?$$

example:

$$\frac{\alpha}{\pi} \ln \frac{m_\tau^2}{m_\rho^2} = 0.004$$

is this a conservative estimate of the uncertainty?

example: $\tau \to \pi\nu$ vs $\pi \to \mu\nu$

affected by intermediate and long distances only

different evaluations differ by about 1%

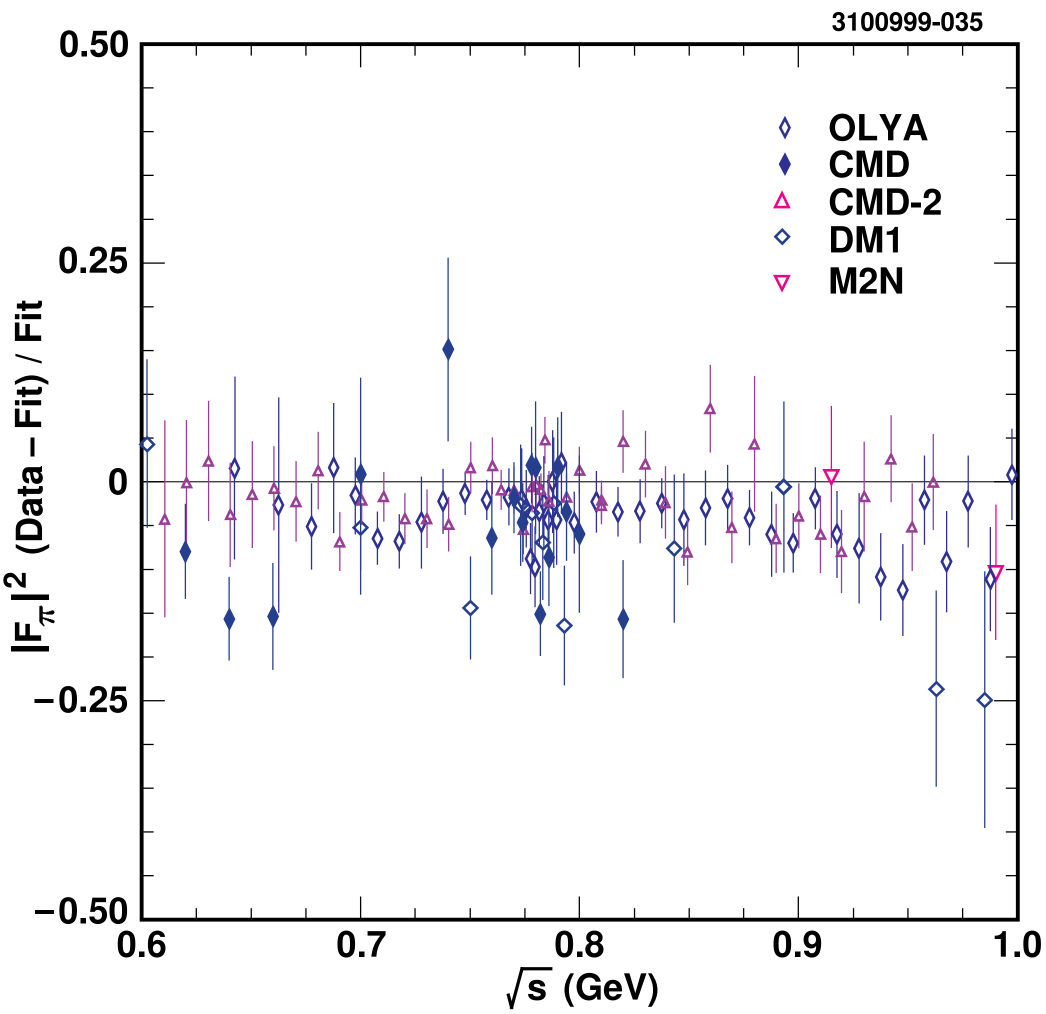
C) kinematic effect: $\pi\pi$

spectral function

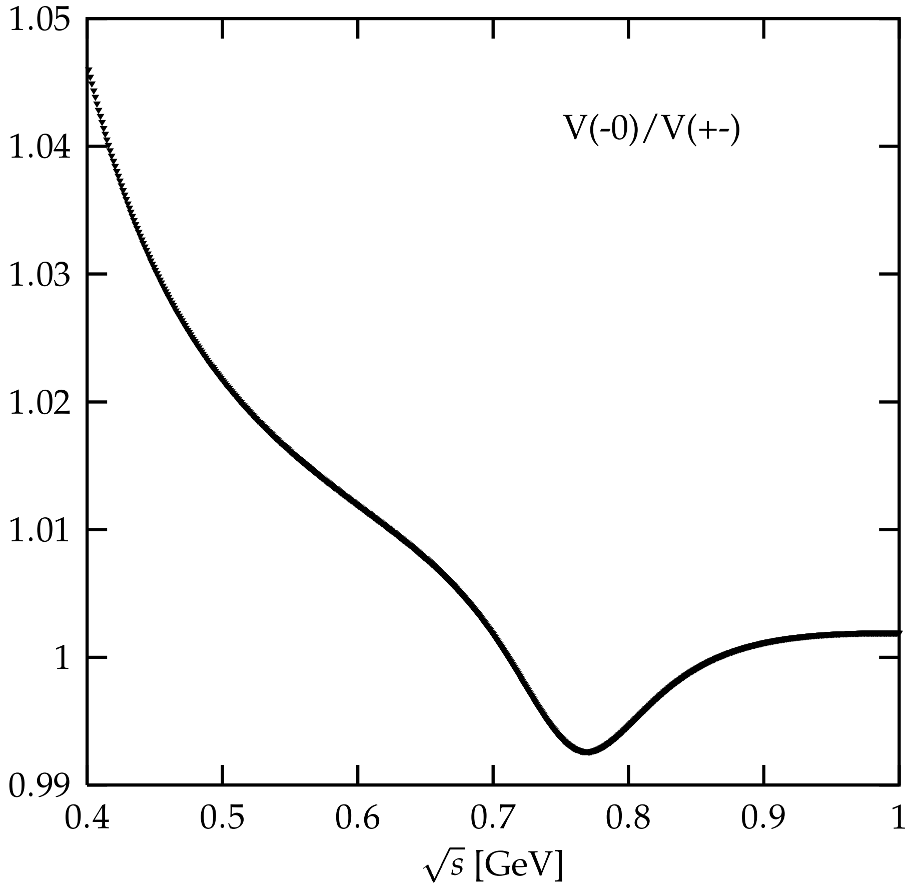
$$v_{\pi\pi} \sim \frac{\Gamma(\rho \to e^+e^-)\Gamma(\rho \to \pi\pi)}{(m_\rho^2 - Q^2)^2 + m_\rho^2\Gamma^2(\rho \to \pi\pi)}$$

$$\Gamma_f \sim \beta^3(m_\pi)$$

experimental distribution from CLEO



[CK, 00]



combined effect for a_μ: -0.58 $\%$ $\hat{=}$ -30 $\times$ 10^{-11}

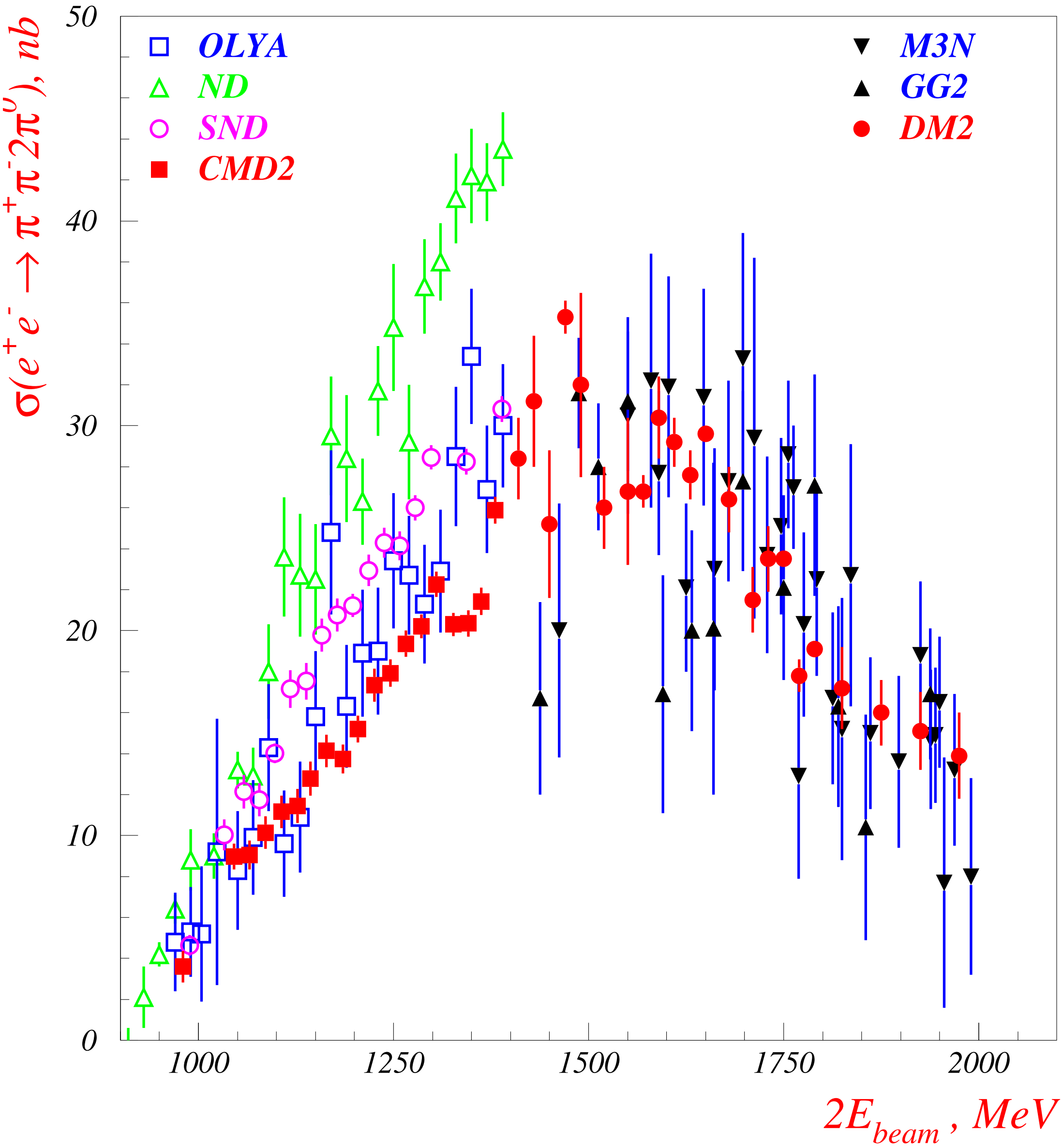
predictions for $\int dQ^2 \frac{d\Gamma}{dQ^2}$ from e^+e^- unaffected

However: most recent evaluation [Eidelman]

$$
\begin{aligned}
B(\pi^-\pi^0) &= 25.31 \pm 0.18 \\
&= 24.94 \pm 0.23 \text{ prediction} \\
& \quad 0.37 \pm 0.29 = \Delta
\end{aligned}
$$

similar (more complicated) problems
with 4π (not yet critical)

➪ precise e^+e^- exp. desirable in the 4π region



$\mathbf{R_{pQCD}}$ below 10 GeV ($\Rightarrow \alpha_s, m_c$)

below charm-threshold

$\sqrt{s}$	α_s	$R_{th}(m_q = 0)$	R_{th}(virt. charm)	R(BES)
2 GeV	0.301	2.204	0.0016	$2.18 \pm 0.07 \pm 0.18$
3 GeV	0.253	2.174	0.0021	$2.11 \pm 0.05 \pm 0.11$
3.7 GeV	0.235	2.161	0.0024	$2.23 \pm 0.08 \pm 0.08$

above charm-threshold

4.8 GeV	0.215	2.145 (u,d,s)		$3.66 \pm 0.14 \pm 0.19$
		1.573 (c)		

pQCD and BES agree amazingly well (2%)

$\langle$ 2 GeV, 3.7 GeV $\rangle \Rightarrow$

$$\alpha_s(M_Z) = 0.144 \pm 0.004 \text{ (stat)} \pm \text{large (syst)}$$

motivates normalization a posteriori

determination of α_s

$$\delta\alpha_s(M_Z^2) \approx \frac{\alpha_s(M_Z^2)}{\alpha_s(Q^2)}\,\delta\alpha_s(Q^2)$$

$\delta R/R = 1\%$ at 2 GeV

would be extremely valuable!!

systematic uncertainties

CLEO (10.52 GeV): $\sim$ 2%
BES (2-3.7 GeV): $\sim$ 5%
MD1 (   ): $\sim$ 4%
DM2 (ρ-region): $\sim$ 1% (now)

significant improvement expected

radiative corrections ?

initial vs final state radiation ?

Sum rules [ITEP] [Rubinstein] [K,Steinhauser]$_{\texttt{preliminary}}$

$$M_n \equiv \int \frac{ds}{s^{n+1}} R_{\texttt{charm}}(s) = \frac{9}{4} Q_c^2 \left(\frac{1}{4m^2}\right)^n C_n$$

$\uparrow$

pQCD

$\mathcal{O}(\alpha_s)$ [Kallen,Sabry]

$\mathcal{O}(\alpha_s^2)$ [CKS]

$\uparrow$

data

$\uparrow$

narrow res+BES+theory($\sqrt{s} > 5$GeV)

results (preliminary)

	M_1	M_2	M_3	M_4	M_5
m_c(3GeV)	1.015	0.988	0.960	0.997	1.086
M_c	1.506	1.47	1.44	1.48	1.58

IV

$\mathrm{R_{had}}$ from tagged photon events

[Binner, Kühn, Melnikov, PLB459 (1999) 279]

[Czyż, Kühn, EPJ C18 (2001) 497]

[Guilleaume, Kühn, Rodrigo, in preparation]

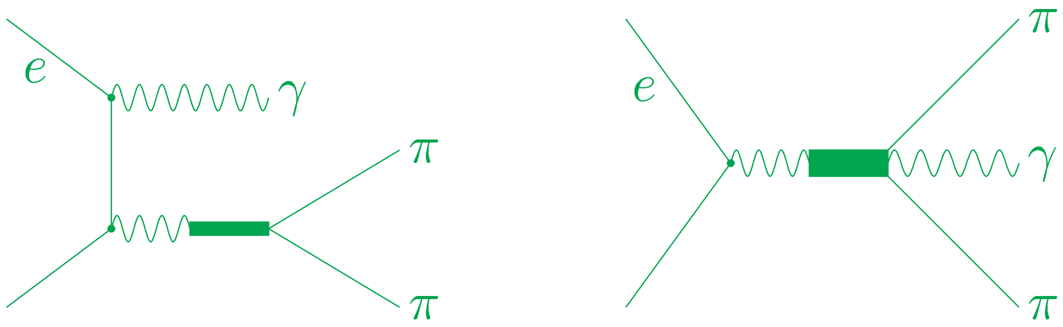
$$e^+ e^- \rightarrow \gamma \pi^+ \pi^-$$

$\pi^+ \pi^- \gamma : E_\gamma > 100 MeV$

$\sqrt{s}\ [GeV]$	$\int \mathcal{L}\ [fb^{-1}]$	#events, $\theta_{min} = 7°$
1.02	1.35	16 $\cdot 10^6$
10.6	100	3.5 $\cdot 10^6$

multi-hadron-events (R $\equiv 2$) $\sqrt{s} = 10.6\, GeV$

Q^2-interval $[GeV]$	#events, $\theta_{min} = 7°$
$[\,1.5\,,\ 2.0\,]$	9.9 $\cdot 10^5$
$[\,2.0\,,\ 2.5\,]$	7.9 $\cdot 10^5$
$[\,2.5\,,\ 3.0\,]$	6.6 $\cdot 10^5$
$[\,3.0\,,\ 3.5\,]$	5.8 $\cdot 10^5$



Solutions:

1) select configuration with dominantly ISR
➪ small angle between γ and $e^{\pm}$

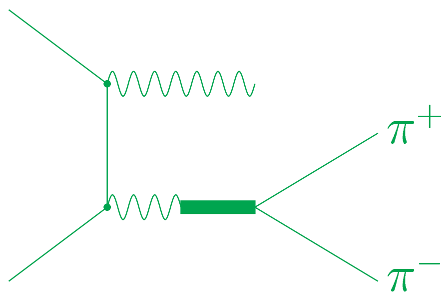
2) allow only configurations where FSR is well predicted ➪ γ soft, collinear with π

3) identify distributions which test model for FSR (angular distributions; charge asymmetry)

quantitative analysis ➪ MC program

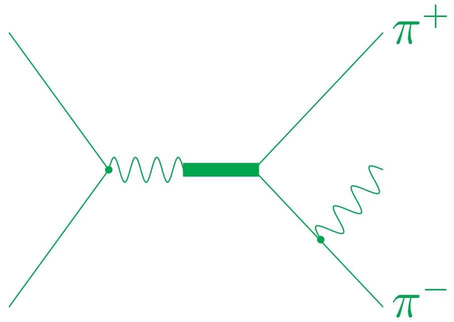
Ingredients:

- ISR



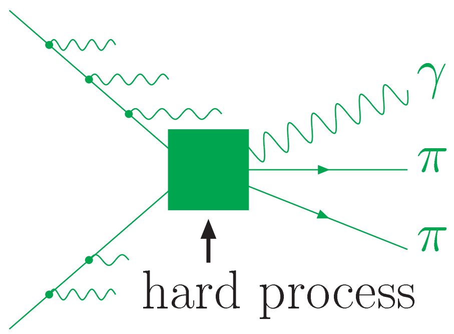
pion form factor
$\rightarrow$ to be tested

- FSR



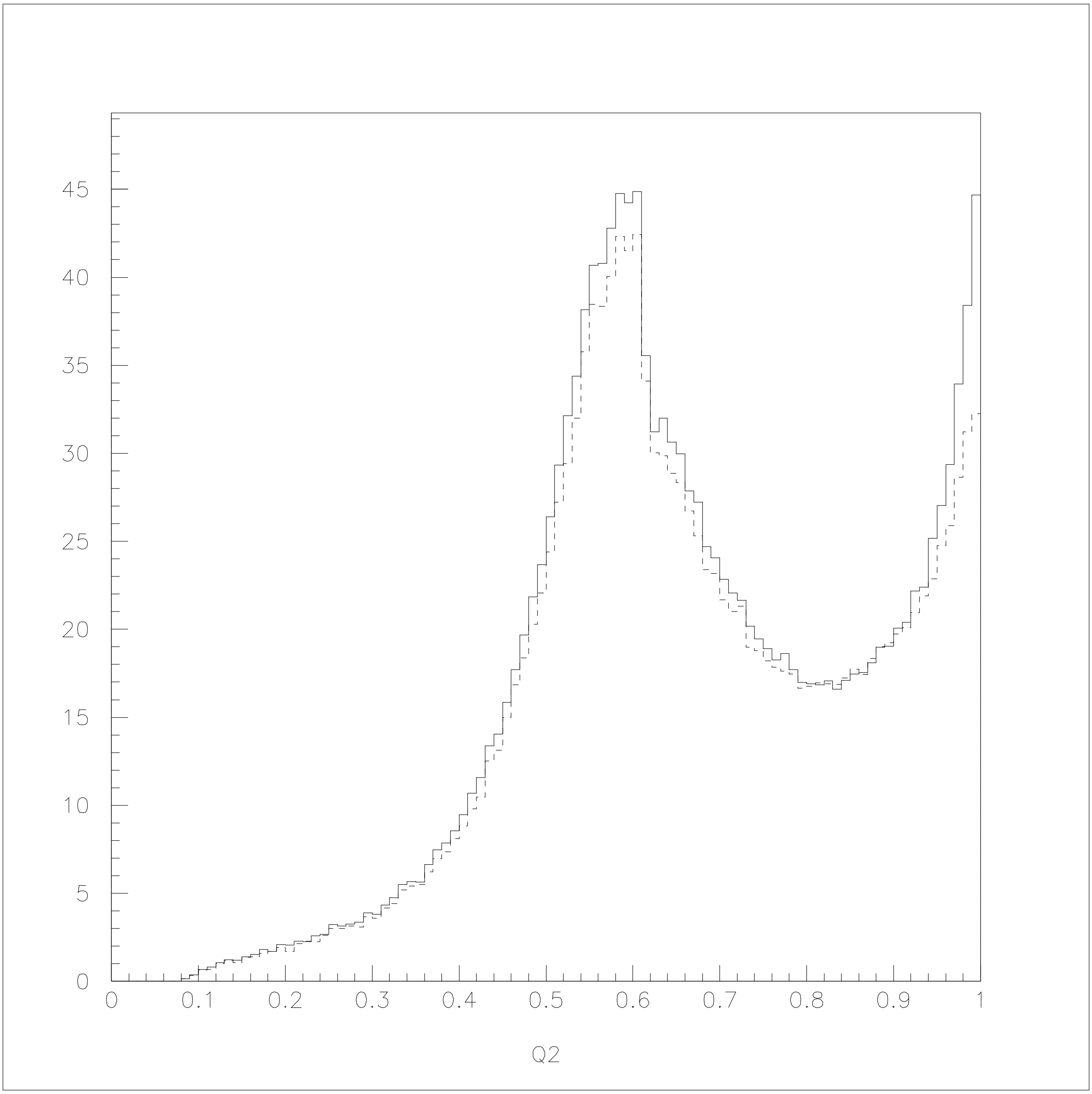
radiation from point-like pions ($\rightarrow$ probably overestimate)

- additional collinear radiation



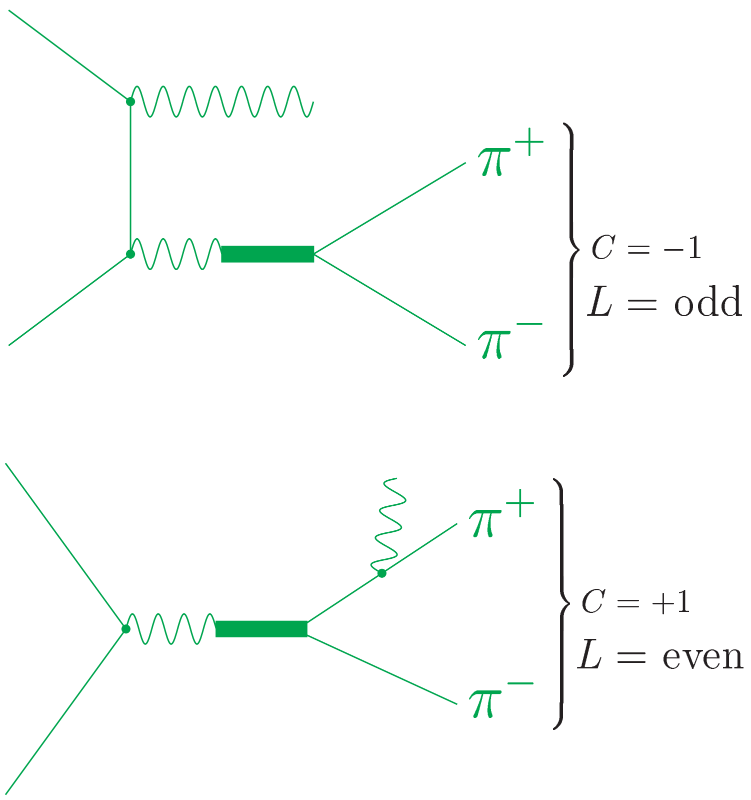
structure function techniques

- $\frac{d\sigma}{dQ^2}$ with / without coll. radiation



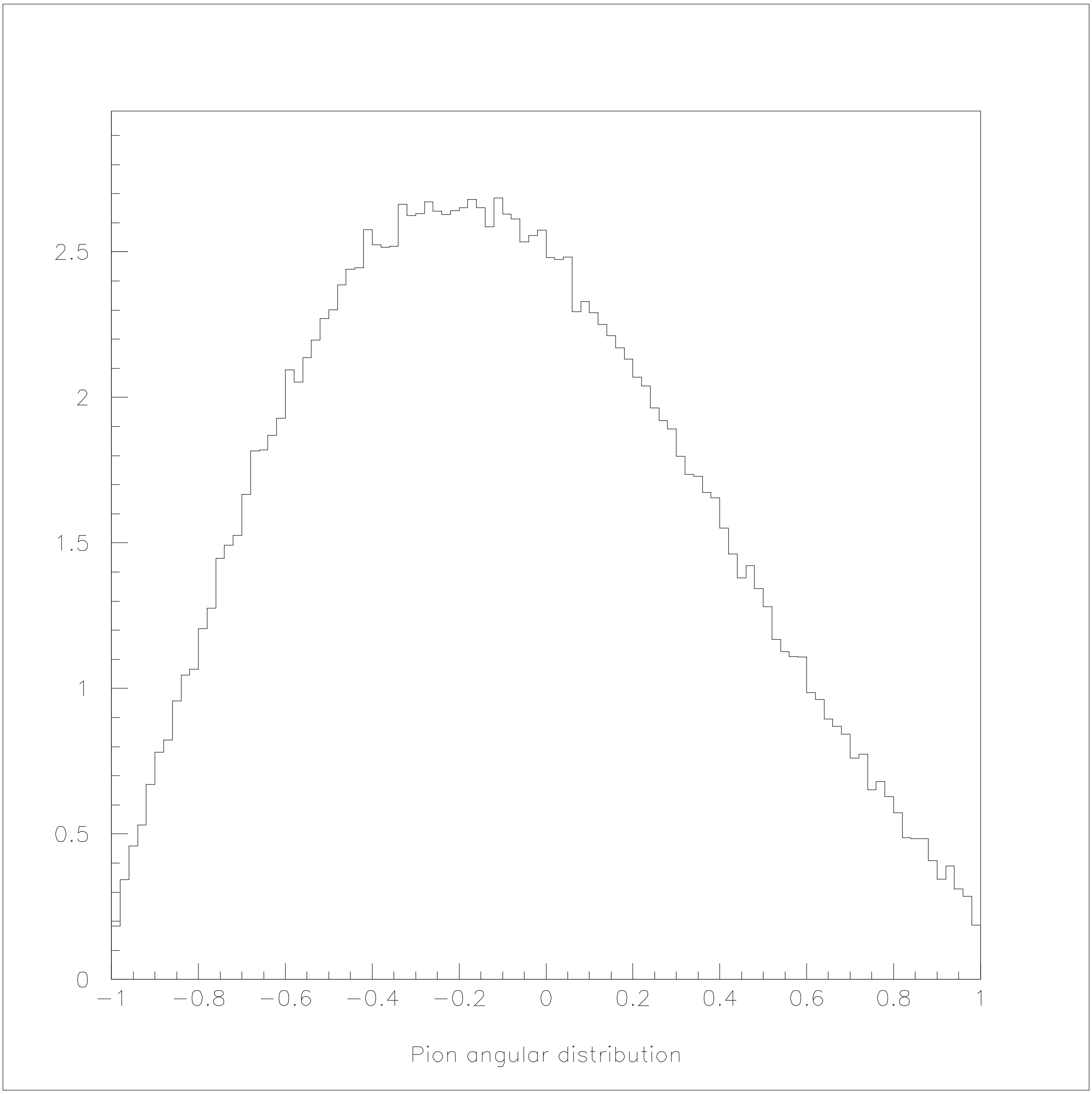
$d\sigma(\, e^+\, e^- \;\rightarrow\; \pi^+\, \pi^-\gamma\,)/dQ^2_{\pi^+\pi^-}$ in nb/GeV^2 vs. $Q^2_{\pi^+\pi^-}$ in GeV^2 for $\sqrt{s} = 1.02\, GeV$, without collinear ISR (solid line) - with collinear ISR (dashed line); for $7^\circ < \theta_\gamma < 173^\circ$ and $Q^2_{\pi^+\pi^-\gamma} > 0.9\; GeV^2$.

interference:



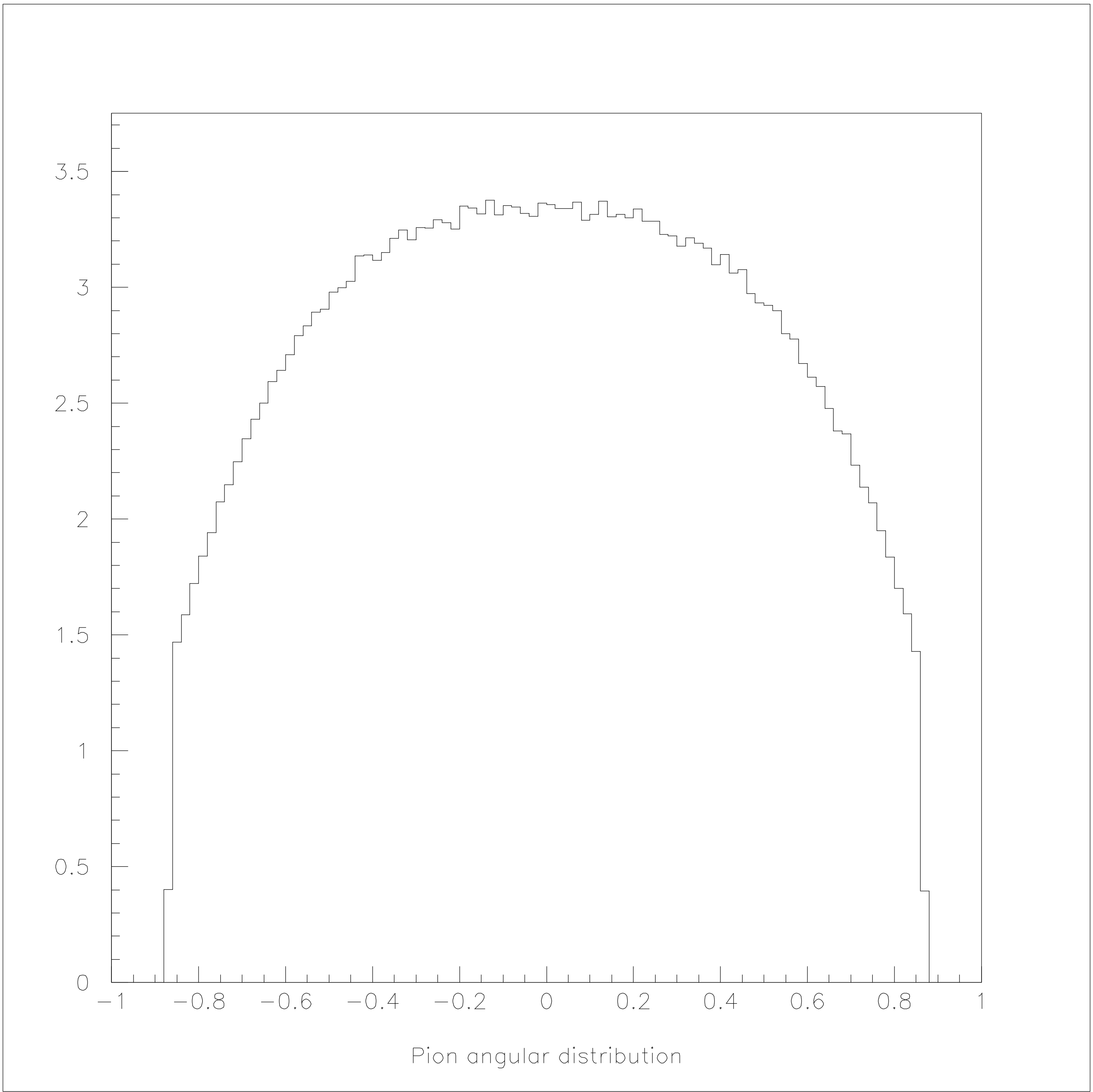
- interference odd under $\pi^+ \leftrightarrow \pi^-$
- asymmetric differential distribution
  $\int$ interf. $= 0$

- Large FB asymmetry of $\pi^{\pm}$ for events with central γ ($\sim 20\%!$)



Pion (π^+) angular distribution in the laboratory frame. The cut on the photon angle is $60° < \theta_\gamma < 120°$ ($\sqrt{s} = 1.02$ GeV).

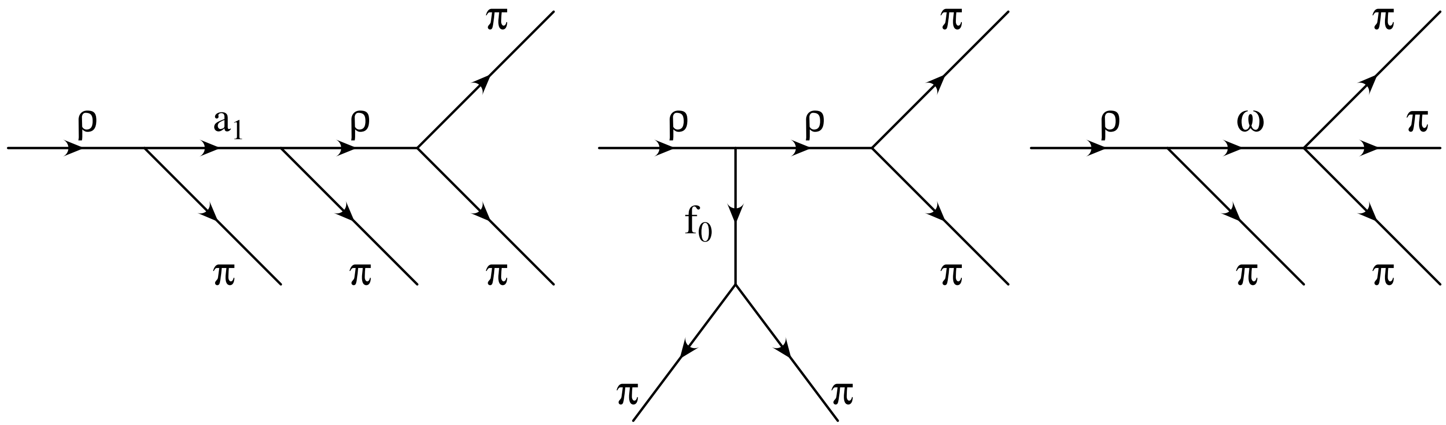
- small FB asymmetry for events with small angle photons and central pions ($\sim 1.5\%$)



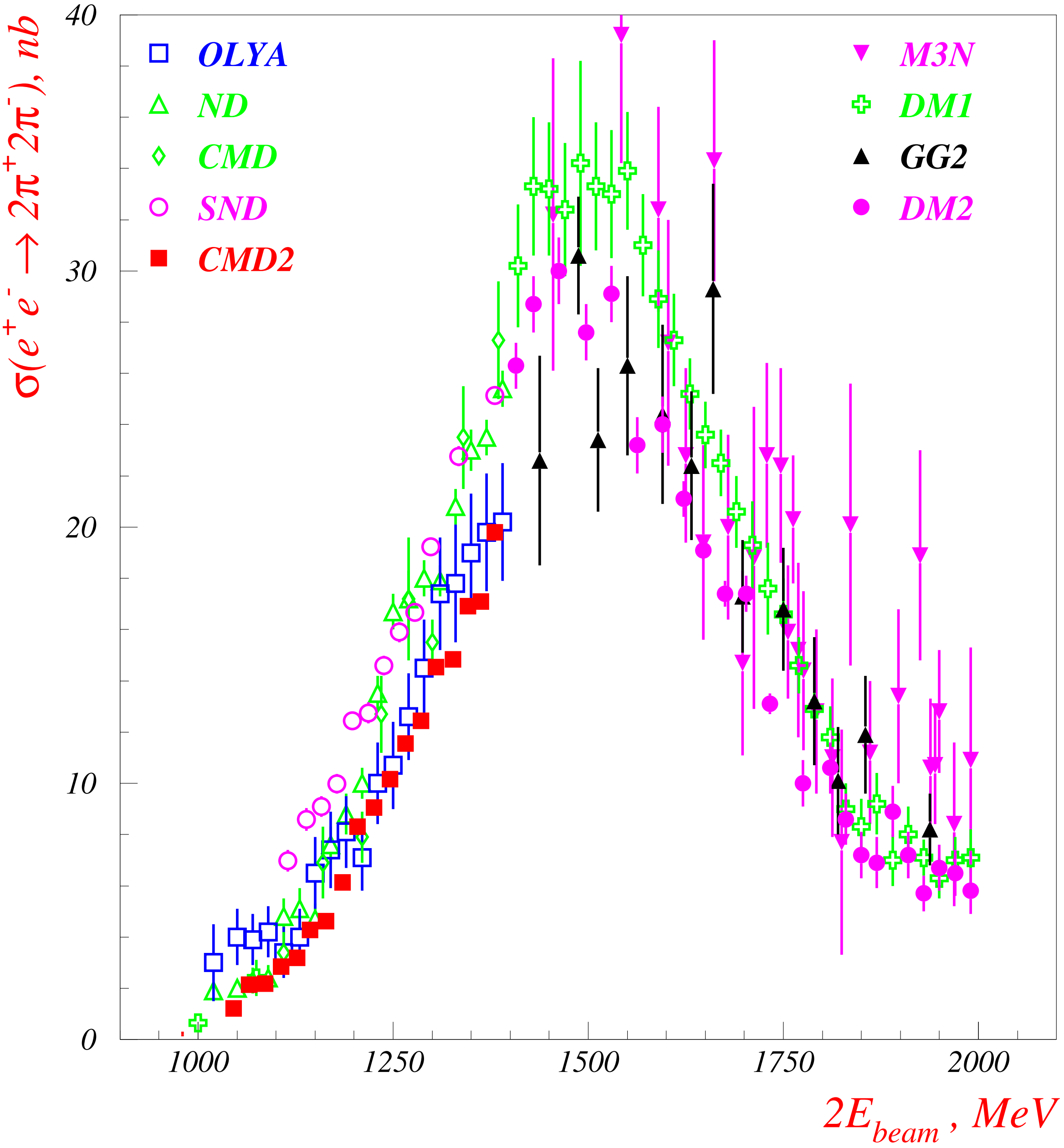
Pion (π^+) angular distribution in the laboratory frame. The photon angle is restricted to $7^\circ < \theta_\gamma < 20^\circ$ or $160^\circ < \theta_\gamma < 173^\circ$. The cut on the pion angle is $30^\circ < \theta_{\pi^+,\pi^-} < 150^\circ$ ($\sqrt{s}$ = 1.02 GeV).

$$e^+ e^- \to \gamma\ 4\pi$$

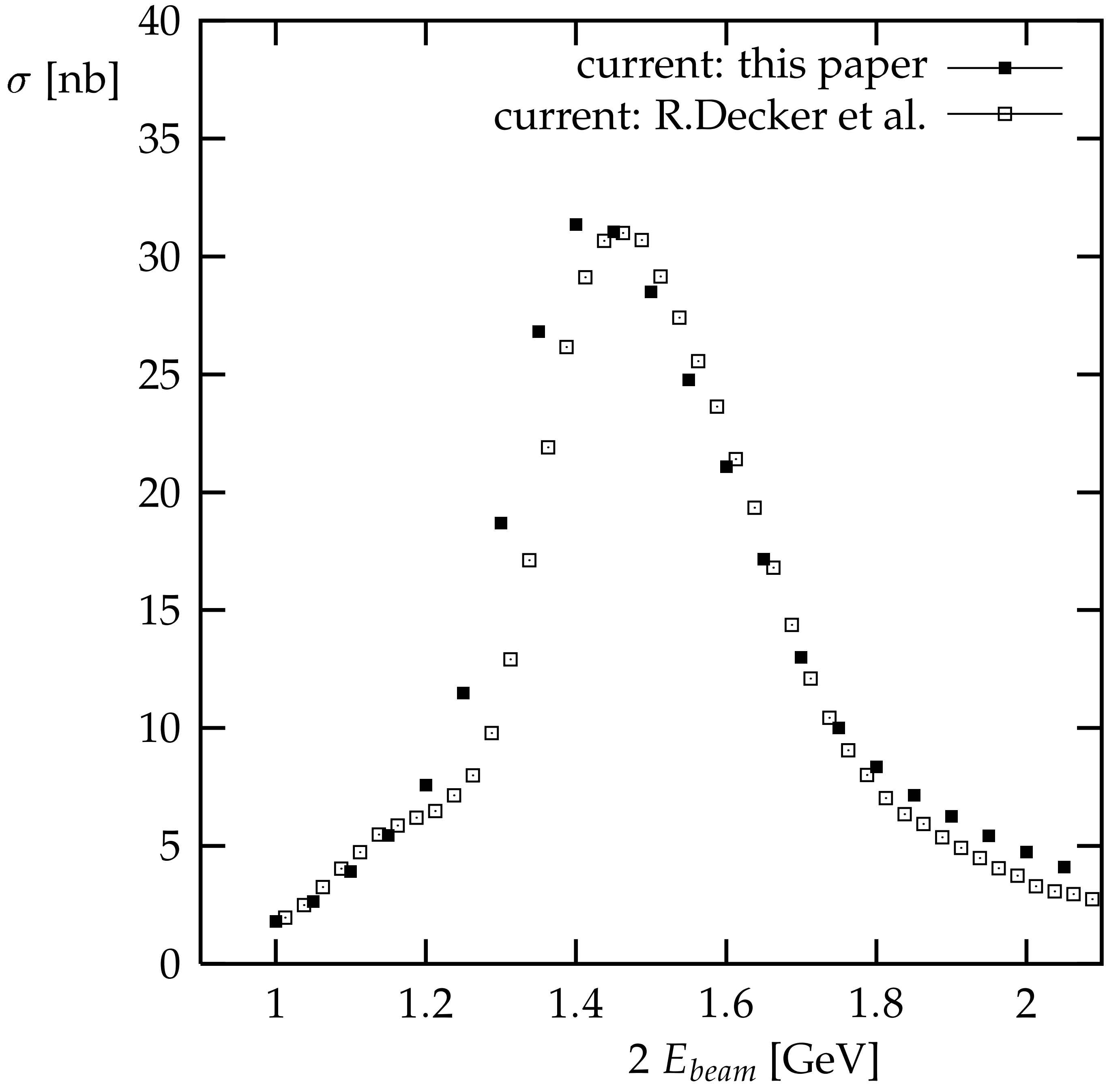
modified/corrected version of model by [Decker *et al.*, ZPC70(247)1996]



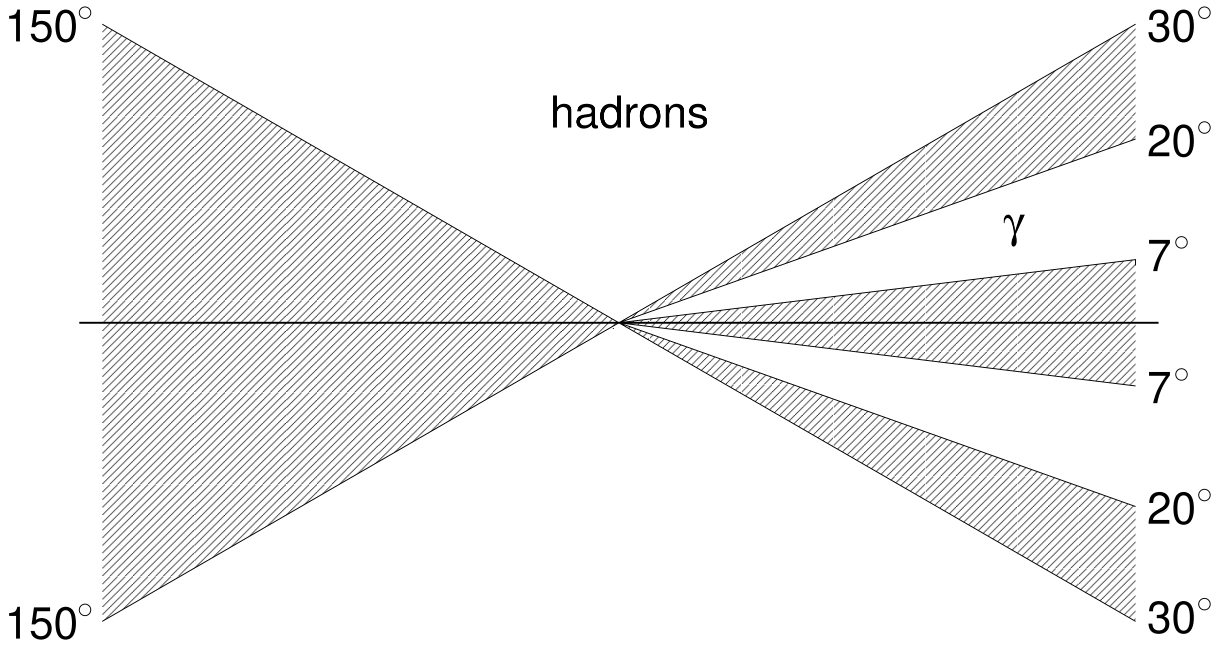
Diagrams contributing to the hadronic current.

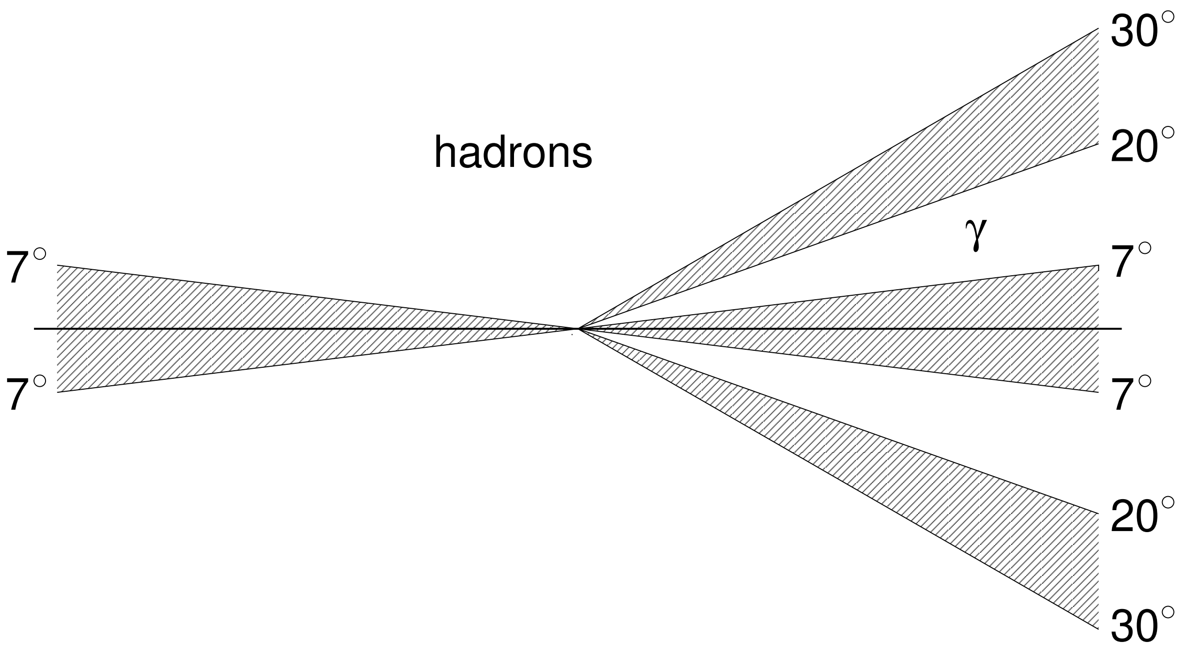


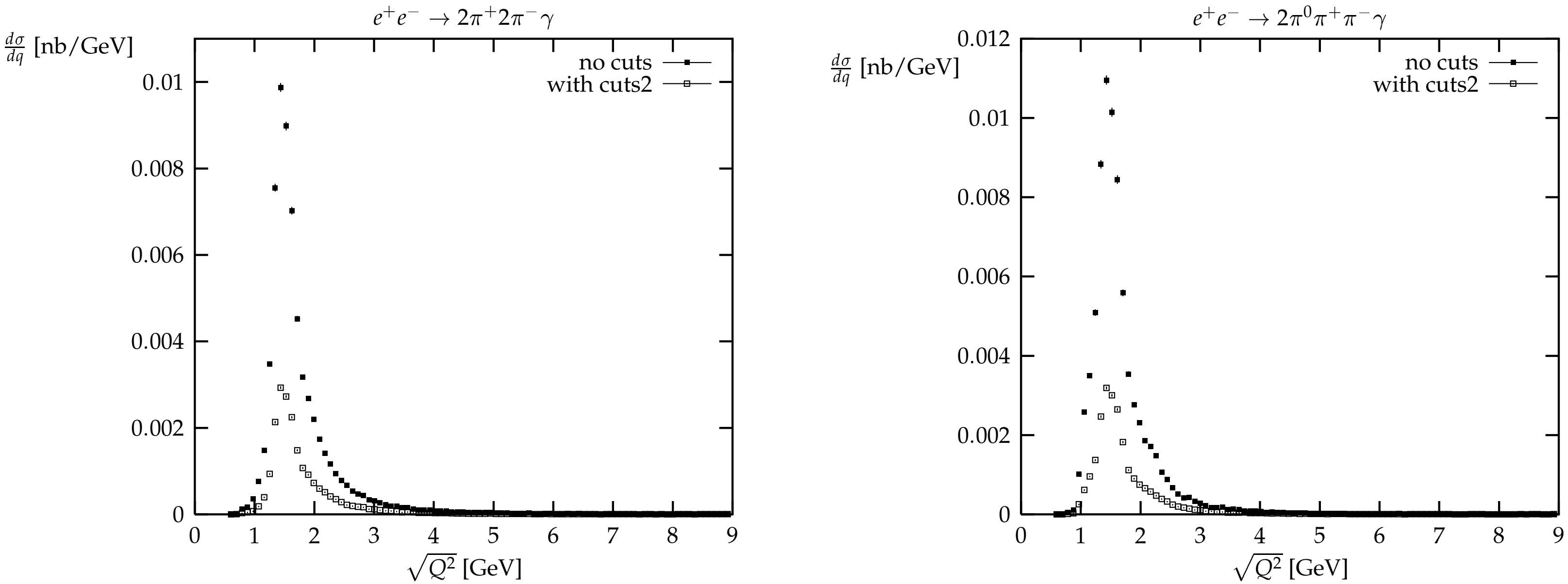
$\sigma(e^+e^- \to 2\pi^+2\pi^-)$, nb
40
30
20
10
0
1000
1250
1500
1750
2000
$2E_{beam}$, MeV
OLYA
ND
CMD
SND
CMD2
M3N
DM1
GG2
DM2



σ [nb]
current: this paper
current: R.Decker et al.
0
5
10
15
20
25
30
35
40
1
1.2
1.4
1.6
1.8
2
2 E_{beam} [GeV]


150°
30°
hadrons
20°
γ
7°
7°
20°
150°
30°


30°
hadrons
20°
γ
7°
7°
7°
7°
20°
30°



The differential $e^+e^- \to 4\pi\gamma$ cross sections at beam energy 5 GeV with minimal photon energy equal 0.2 GeV with no angular cuts (no cuts) and angular cuts (with cuts2), where $q \equiv \sqrt{Q^2}$ is the invariant mass of the 4π system.

$E_{Beam} = 5GeV$, $E_\gamma > 200$ MeV

$$\sigma(e^+e^- \to 2\pi^+\, 2\pi^-\, \gamma) \;=\; 0.00592\, nb$$
$$cuts2: \;=\; 0.00186\, nb$$
$$\sigma(e^+e^- \to 2\pi^0\pi^+\pi^-\gamma) \;=\; 0.00711\, nb$$
$$cuts2: \;=\; 0.00217\, nb$$

$$N_{ev} \sim (6-7)\, 10^4$$
cuts 2 : $*\frac{1}{3}$

collinear radiation ➪ reduction by 10 %

V

SUMMARY

- systematics ?

  theory: higher orders

  expt. ?

- FSR can be controlled through FB-asymmetry and proper cuts

- Generators for $e^+e^- \to 2\pi\gamma$ and $e^+e^- \to 4\pi\gamma$ exist.

- The 4π reaction is accessible at DAPHNE for a limited region of phase space. Interesting possibility for B-factories!