Initial Radiative Correction for R Measurement at BES

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Abstract Three typical schemes of the initial state radiative correction in e^+e^- collision producing hadrons through single photon annihilation were studied, and the numerical results were calculated in the energy region $2-5$ GeV. The uncertainty of theoretical values $(1+\delta)$ is smaller than 1% below 3 GeV, and about $1-1.5\%$ in resonant region. The uncertainty of effective $(1+\delta_{obs})$ is also estimated from some aspects related to experiments setting. The differences of the radiative correction factors $(1+\delta_{obs})$ are about $1-1.5\%$ below $3GeV$ and about $2 \sim 3.5\%$ in the resonant region.

1 Introduction

R is defined as the lowest order hadronic cross section in e^+e^- collision via the single-photon annihilation normalized by $\mu^+\mu^-$ pairs

$$R = \frac{\sigma^{\circ}_{tot}(e^+e^- \to \gamma^* \to hadrons)}{\sigma^{\circ}(e^+e^- \to \gamma^* \to \mu^+\mu^-)}. \tag{1}$$

In experiment, R is determined by the following formula

$$R = \frac{N^{obs}_{had} - N_{bg}}{\sigma^{\circ}_{\mu\mu} \cdot L \cdot \epsilon_{had} \cdot (1+\delta)}. \tag{2}$$

where $(1+\delta)$ is the initial state radiative (ISR) correction factor which represents the contributions of all higher-order Feynman diagrams (Fig.1) in the measured cross section σ^{obs}_{had}. The final state radiative (FSR) correction concerns the nonperturbative hadronization calculations for which we can only get the model-dependent results. R value measures the inclusive hadronic cross section which includes the summation over the contributions of all hadronic final states, KLM theorem has proved that the final state radiative corrections canceled strongly[1][2][3], and their contribution to $(1+\delta)$ less than 1%, and therefore we ignored it in our calculations at present experiment precision.

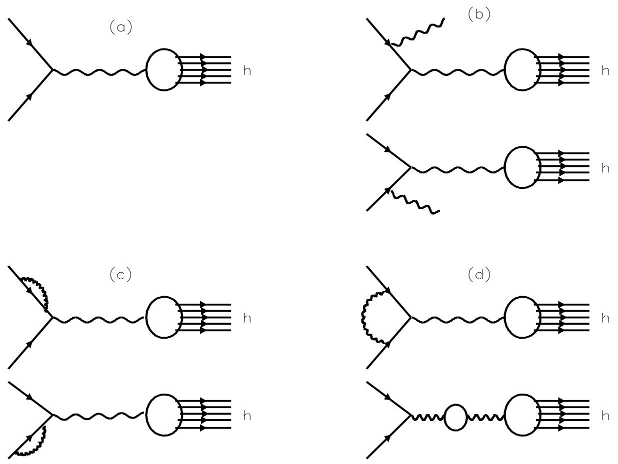

图 **1 :** Feynman of $e^+e^- \Rightarrow$ hadrons. (a)lowest order figure; (b)initial bremsstrahlung; (c)e^+ and e^-self-energy figure; (d) initial vertex and vacuum polarization figures. Radiative correction factorδ represents the total contribution of figures (b), (c) and (d).

2 Some discussions related to experiment setting

2.1 The general consideration about hadronic cross sections

For a selected radiative correction scheme (see next section), one of the important question is to analysis the lowest level cross sections. If we assume that no interference between continuous and resonant states, the lowest order and the total cross sections may be represented as following formulas,

$$\sigma^\circ(s) = \sigma^\circ_{con}(s) + \sigma^\circ_{res}(s), \tag{3}$$
$$\sigma^T(s) = \sigma^T_{con}(s) + \sigma^T_{res}(s). \tag{4}$$

The relations between σ° and σ^T is defined as

$$\sigma^T \equiv \sigma^\circ(1+\delta). \tag{5}$$

The lowest level continuous cross section $\sigma^\circ_{con}(s)$ may be expressed by

$$\sigma^\circ_{con}(s) = \tilde{R}(s) \cdot \sigma^\circ_{\mu\mu}(s), \tag{6}$$

$\tilde{R}$ is the R value in which the resonant contribution was removed. From hadronic threshold $\sqrt{s} = 2m_\pi$ to 1.8 GeV energies, $\tilde{R}$ was use the fitted experiment values $\tilde{R}_{exp}$, for $\sqrt{s} > 1.8$ GeV energies, the precision of R_{QCD} in perturbative QCD is better than experiment values[4], and R_{QCD} and $\tilde{R}_{exp}$ are consistent under the present errors, so $\tilde{R}$ was replaced with R_{QCD}.

There are abundant resonant states in BEPC energies region, the structures of resonant states will appear in the radiative correction factors. In our calculations, 14 resonances of which masses less than $4.5GeV$ such as

$$\rho,\ \omega,\ \phi,\ J/\psi,\ \psi\text{'},\ \psi(3770),\ \cdots\ ,$$

were accounted. The resonant cross section for hadronic final state is given by the relativistical Breit-Wigner form (RL)

$$\sigma_{BW}(s) = \frac{12\pi\Gamma^{ee}\Gamma^{had}}{(s-M^2)^2+(M\Gamma^{tot})^2}, \tag{7}$$

or nonrelativistical form (NRL)

$$\sigma_{BW}(s) = \frac{3\pi}{s}\cdot\frac{\Gamma^{ee}\Gamma^{had}}{(W-M)^2+(\Gamma^{tot}/2)^2}. \tag{8}$$

If we ignore the interferential effects between the resonances, the total lowest level cross section reads

$$\sigma^{\circ}_{res}(s) = \sum \sigma_{BW}(s). \tag{9}$$

When the energy at which R is measured far from resonant peaks, the detailed resonant structures are not important, and therefore the Breit-Wigner cross section may be simplified effectively as δ form (except for the wide resonances),

$$\sigma_{BW}(s) \ \Rightarrow\ 12\pi^2\frac{\Gamma^{ee}\Gamma^{had}}{\Gamma^{tot}M}\delta(s-M^2), \qquad (RL), \tag{10}$$

$$\sigma_{BW}(s) \ \Rightarrow\ \frac{6\pi^2}{M^2}\frac{\Gamma^{e}\Gamma^{had}}{\Gamma^{tot}}\delta(W-M), \qquad (NRL). \tag{11}$$

There will be e^+e^- beam energy-spread for BEPC, the distribution may be written as Gaussian form

$$G(W-W') = \frac{1}{\sqrt{2\pi}\Delta}\exp\left[-\frac{(W-W')^2}{2\Delta^2}\right], \tag{12}$$

where, $W=\sqrt{s}=2E$ is the nominal energy of e^+e^- in storage ring, $W'=\sqrt{s'}$ is the actual energy, and Δ is the variance of W. At BEPC, the energy spread Δ is dependent on the W and the electric current I,

$$\Delta(MeV) = [0.00722I(mA)+0.4780][0.0705W^2(GeV)+1.0656]. \tag{13}$$

For the energies near the resonant peaks, the energy-spread will affect the resonant cross section. The effective cross section $\tilde{\sigma}_{res}$ may be represented as a integration

$$\tilde{\sigma}^T_{res}(s) = \int_0^{+\infty} dW'\sigma^T_{res}(s')G(W-W'). \tag{14}$$

The above integration can also calculate by Monte Carlo method,

$$\tilde{\sigma}^T_{res}(s) = \frac{1}{N}\sum_{i=1}^{N}\sigma^T_{res}(s^{'}_i), \tag{15}$$

where $(s_1^{'}, s_2^{'}, \cdots, s_N^{'})$ is a large random sample with the energy-spread distribution $G(W - W')$.

The relations between the observed hadronic production cross sections in experiment $\sigma_{obs}(s)$ and the physical (theoretical) cross sections $\sigma(s)$ are

$$\sigma_{obs}^{\circ}(s) = \epsilon^{\circ} \cdot \sigma^{\circ}(s), \tag{16}$$

$$\sigma_{obs}^{T}(s) = \bar{\epsilon} \cdot \sigma^{T}(s), \tag{17}$$

where, ϵ° and $\bar{\epsilon}$ are the hadronic efficiencies with and without initial state radiation and they may estimated by Monte Carlo.

2.2 The energy-dependence of wide resonant width

In BEPC energy region, $\psi(3770)$, $\psi(4040)$, $\psi(4140)$ and $\psi(4410)$ may be viewed as wide resonances. At present, the decay channels of these resonances were poorly understood in theory and experiments. According to the predictions of Eichten[5] model there are following decay channels (include their charge conjugate states)

$$\begin{aligned}
\psi(3770) &\Rightarrow D\bar{D}; \\
\psi(4040) &\Rightarrow D\bar{D}, D^*\bar{D}^*, D\bar{D}^*, \bar{D}D^*, D_s\bar{D}_s; \\
\psi(4140) &\Rightarrow D\bar{D}, D^*\bar{D}^*, D\bar{D}^*, \bar{D}D^*, D_s\bar{D}_s, D_s\bar{D}_s^*; \\
\psi(4415) &\Rightarrow D\bar{D}, D^*\bar{D}^*, D\bar{D}^*, \bar{D}D^*, D_s\bar{D}_s, D_s\bar{D}_s^*, D_s^*\bar{D}_s^*.
\end{aligned}$$

Each decay channel is characterized by its decay width Γ_i. Sum over all the decay channels, the total decay width of resonance is

$$\Gamma^{tot}(s) = \sum_{ch} \Gamma_{ch}^{tot}(s). \tag{18}$$

According to the quantum mechanics, the total decay width of wide resonance is dependent on the momentum and quantum number L of orbital angular momentum of the final states[6]. The energy dependence of $\Gamma(s)$ contains a phase space factor (which is the function of decay momentum in its c.m.s.) and a centrifugal barrier factor B_L. The later depends on the nature of the interaction, but for the first approximation, the expression of B_L may be derived for a square well potential for nonrelativistic energies in quantum mechanics. We need analysis and sum all the possible decay partial waves and then obtain the following expression for total width

$$\Gamma_{ch}^{tot}(s) = \Gamma_R \sum_{L} \frac{z^{2L+1}}{B_L}, \tag{19}$$

where, $z \equiv rp$. r is the radius of interaction and is order of $r \sim 1 - 3$ fermi which is estimated according to the strong interaction region. Factor Γ_R may be obtained by data fitting. p is the momentum of final state particle in its c.m. system,

$$\begin{aligned}
p^2 &= \lambda^2(s, M_{D_1}^2, M_{D_2}^2)/4s \\
&= \frac{[s - (M_{D_1} + M_{D_2})^2][s - (M_{D_1} - M_{D_2})^2]}{4s}.
\end{aligned} \tag{20}$$

The first few energy-dependent partial wave functions B_L are [7]

$$\begin{aligned}
B_0 &= 1, \\
B_1 &= 1 + z^2, \\
B_2 &= 9 + 3z^2 + z^4, \\
B_3 &= 225 + 45z^2 + 6z^4 + z^6.
\end{aligned} \tag{21}$$

The decay partial wave is proportional to p_{ch}^{2L+1}, this effect is called as centrifugal barrier. When the decay momentum p is not large (near the production threshold), the partial decay widths decrease rapidly with the L increase. This property causes the unsymetry of Breit-Weigner cross section (staring from the peak position, $\sigma_{BW}(W)$ decreases more rapidly when W decrease than W increase). In the case of some decay channel may occurs in several partial waves, the lower partial waves are important. A relativistic correction factor which accounts to $2M_R/(M_R + W)$ may be introduced in $\Gamma^{tot}(s)$

$$\Gamma_{ch}^{tot}(s) = \Gamma_R \cdot \frac{2M_R}{M_R + W} \sum_L \frac{z^{2L+1}}{B_L}, \tag{22}$$

2.3 The coherence of the resonances

In the ψ-family resonant region, there is not same hadronic final state for continuous and resonant decays, and therefore we may assume that no coherence between them (i.e. assume that all two-body DD states are decay products of ψ-family resonances, not produced directly in continuous hadronization mechanism). But for the four wide resonances, they have the same decay final states, and their peaks are close and they appear significant overlap which lead to the interferences. Therefore the amplitudes corresponding to each resonance have to be added coherently to give the total amplitude which, once squared, will contain interferences of the type $ReT_i^*T_j$. If the resonances are quite broad, the interference effect can also distort the resonance shape, the width might appear broader or narrower, and the position of the peak can be displaced. In our experiments, ψ-family resonances produce at the energies which are near their threshold, and relativistical effect is weak. We may use the nonrelativistical resonant Breit-Wigner amplitude

$$T_R = \frac{\pm\frac{1}{2}\sqrt{\Gamma^{ee}\Gamma^{had}}}{W - M + i\Gamma^{tot}/2}, \qquad (NRL). \tag{23}$$

In relativistical quantum mechanics, we should consider the contribution of anti-particle (i.e. particle with negative energy). It is easy to extend formula (23) to relativistical amplitude

$$T_R = \frac{\pm M\sqrt{\Gamma^{e}\Gamma^{had}}}{s - M^2 + iM\Gamma^{tot}}, \qquad (RL). \tag{24}$$

The total resonant cross section of ψ-family is given by

$$\sigma^{\circ}_{res} = \frac{4\pi(2J+1)}{s} |\sum_R T_R|^2. \tag{25}$$

If the difference of two peak $|M_{R_1} - M_{R_2}| \gg \Gamma^{tot}_{R_1} + \Gamma^{tot}_{R_2}$, their interferential effects may be ignored. Fig.2 shows the changes of R values of without interference and interferential terms and total ones of $\psi(4040), \psi(4140)$ and $\psi(4415)$ with energies. The interferential effects are clear, and they lead the unsymmetry on lowest level resonant cross section

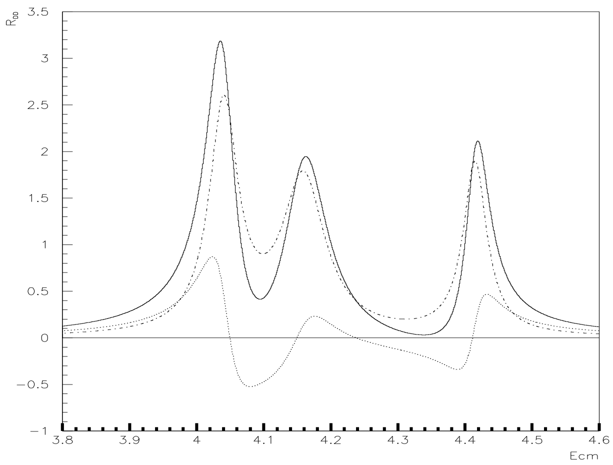

图 **2 :** The R values from $\psi(4040), \psi(4160)$ and $\psi(4416)$, the dot-line for noninterferential term, the dashed-line for interference term and the solid-line for total ones, in which we use the resonant parameters in PDG

2.4 The soft photon questions

The initial state radiative correction may be divided into two classes, one is the real photon emission and another is the virtual photon emission. The real photon radiations low the effective energies of using in hadrons production. When initial e^+ or e^- emit a real photon with energy $k \equiv E_\gamma/E$, the effective c.m. energy in hadron production is

$$s' = s(1-k). \tag{26}$$

It has been proved that for each fixed order $O(\alpha^n)$, QED is renormalizable if all corresponding Feynman diagrams are combined, by which the ultrared divergences of loop diagrams can be removed. The PQED corrections in both virtual and real photon emission are infrared divergencies. The divergence in real photon emission comes from photon with zero energy, and come from photon with zero mass and energy in virtual photon part . One skill to solve this problem is to introduce a small photon mass λ in order to keep the theory is Lorentz invariant and to regularize the infrared divergences, and an arbitrary value k_0 for describing separately the infrared behavior for soft bremsstrahlung and the nonfactorizing cross section for hard bremsstrahlung. The infrared divergence in the virtual corrections is just an artefact of perturbative theory. Since the zero energy and soft photon are not observable, if we define[8]

$$\sigma^{VSB} \equiv \sigma_\circ + \sigma^V + \sigma^{SB} = \sigma^\circ(1 + \delta^V + \delta^{SB}), \tag{27}$$

$$\sigma^H \equiv \sigma^{HB}, \tag{28}$$

where, σ^V is the cross section for virtual radiations, σ^{SB} for the soft bremsstrahlung. λ containing in σ^{VSB} was canceled, i.e. it is λ-independent. Both σ^{VSB} and σ^H are finite, but in its (λ) place a dependence on k_0 left. In the approximation of single photon emission, the cross section with hard photon emission correction may be obtained by photon spectrum distribution[9]

$$\frac{\partial\sigma^{HB}}{\partial k} = \frac{\beta}{k}(1 - k + \frac{1}{2}k^2)\sigma^\circ(s'), \tag{29}$$

or

$$\sigma^H = \int_{k_0}^{k_m} dk \frac{\partial\sigma^{HB}}{\partial k}. \tag{30}$$

Where,

$$\beta = \frac{2\alpha}{\pi}[\ln\frac{s}{m_e^2} - 1], \tag{31}$$

which appears at the each vertex of bremsstrahlung, and is called as effective bremsstrahlung coupling-constant. As $k_0 \to 0$, $\sigma^S \to -\infty$, there is some k_c for k_0, at which σ^S becomes negative. Reasonable k_0 values keep $\sigma^S > 0$. Cutoff k_0 is not a physical quantity. In physics, the calculation results should not depend on the choice of k_0 within reasonable limits. The following requirements are considered: (1) upper limit: it is imposed by phenomenological considerations, k_m may be determined according to the effective c.m. energy must be larger than the threshold of hadron production. (2)lower limit: reasonable k_0 values keep $\sigma^S > 0$.

From the point of view of experiment, the arbitrary of division of soft and hard photon may be fixed by connecting k_0 with the resolution of detector. Only those real bremsstrahlung photon with $k > k_0$ were observed, and those with $k < k_0$ have to be "lumped" together with the virtual photons. This means that k_0 should at least be smaller than the resolution power of the detector. The minimum photon energy observed in BES is about $50 \sim 80$ MeV, which corresponding to $0.01 < k_0 < 0.05$.

In the estimation of hadronic efficiencies by Monte Carlo, events produce according to the cross section (53) and (28). The events corresponding to σ^{VSB} were treat as no-emission one, and the events corresponding to σ^H produce with the bremsstrahlung distribution(29). The arbitrariness of k_0 will leads to the uncertainty of hadronic efficiencies and initial state radiation factors.

The calculations of perturbative QED show that for the soft photon ($k < k_0$) emission the contributions from higher level Feynman figures are important. The more precise calculation should be including the contributions from the figure with any number of soft photons emission. Multi-soft photons cross sections have the leading form [10]

$$\begin{aligned}\sigma^{S_1 S_2 S_3 \cdots} &= \sigma^{\circ} \times [1+\beta \ln k_0 + \frac{1}{2}\beta^2 \ln^2 k_0 + \frac{1}{3!}\beta^3 \ln^3 k_0 + \cdots] \\ &= \sigma^{\circ} \times exp(\beta \ln k_0) = \sigma_0 \times k_0^{\beta},\end{aligned} \tag{32}$$

i.e., the total contribution is that the soft photon cross sections have the exponential property, which is equivalent to introduce directly a factor k^β in single photon bremsstrahlung distribution $\partial\sigma^{HB}/\partial k$, the bremsstrahlung spectrum becomes

$$\frac{\partial \sigma}{\partial k} = k^{\beta} \cdot \frac{\partial \sigma^{HB}}{\partial k}, \tag{33}$$

Its integration on the interval $[0, 1]$ is convergence, and the so-called k_0 problem was removed.

2.5 The Vacuum polarization

The contribution of vacuum polarization change the photon propagator in flowing way

$$\frac{-ig_{\mu\nu}}{q^2} \Rightarrow \frac{-ig_{\mu\nu}}{q^2[1-\Pi(q^2)]}. \tag{34}$$

To α leading level, the renormalized lowest cross section becomes to

$$\sigma^{\circ}(s) \Rightarrow \frac{\sigma^{\circ}(s)}{|1-\hat{\Pi}_2(s)|^2}, \tag{35}$$

in which, $\hat{\Pi}_2(s) \equiv \Pi_2(s) - \Pi_2(0)$ is finite , and its real part may be expressed by dispersion relation

$$Re\hat{\Pi}_2(s) = \frac{s}{4\pi^2\alpha} \sum_X \int_{s_{th}}^{\infty} \frac{\sigma_{ee\to X}(s')ds'}{s-s'-i\epsilon}. \tag{36}$$

The sum over X is for the fermion loop in vacuum polarization may be $X = e^+e^-, \mu^+\mu^-, \tau^+\tau^-, q\bar{q}$ pairs. When $\hat{\Pi}_2(s)$ is small, it can be expanded in series and take only the first two terms

$$\frac{1}{|1-\hat{\Pi}_2(s)|^2} = 1 + 2Re\hat{\Pi}_2(s) + \cdots. \tag{37}$$

This means that the total vacuum polarization factor may be written as the sum of each term

$$\delta_{vac} = -2Re\hat{\Pi}_2(s) = \delta_{vac}^{e^+e^-} + \delta_{vac}^{\mu^+\mu^-} + \delta_{vac}^{\tau^+\tau^-} + \delta_{vac}^{q\bar{q}}. \tag{38}$$

For lepton-loop, the dispersion integration may be calculated analytically

$$\delta_{vac}^{l^+l^-}(s) = \frac{2\alpha}{\pi} f(x), \quad (x = 4m_l^2/s), \tag{39}$$

where,

$$f(x) = -\frac{5}{9} - \frac{x}{3} + \frac{\sqrt{1-x}(2+x)}{6} \log\left[\frac{1+\sqrt{1-x}}{1-\sqrt{1-x}}\right], \quad (x \le 1), \tag{40}$$

$$f(x) = -\frac{5}{9} - \frac{x}{3} + \frac{\sqrt{x-1}(2+x)}{3} \tan^{-1}\frac{1}{\sqrt{x-1}}, \quad (x > 1). \tag{41}$$

There are two ways to calculate the term of quark-antiquark loop:
(A)corresponding effectively $q\bar{q}$ to hadronic states the correction factor may be subdivided into the sum of the contributions of continuous and resonant states

$$\delta_{vac}^{q\bar{q}} \equiv \delta_{vac}^{had}(s) = \delta_{vac}^{res}(s) + \delta_{vac}^{con}(s), \tag{42}$$

where,

$$\delta_{vac}^{res}(s) = \sum_j \frac{6\Gamma_j^{ee}}{\alpha M_j} \cdot \frac{s\left[s - M_j^2 + \Gamma_j^{tot} M_j \pi^{-1} ln(s/m_e^2)\right]}{\left(s - M_j^2\right)^2 + \left(M_j \Gamma_j^{tot}\right)^2} \tag{43}$$

$$\delta_{vac}^{con}(s) = -\frac{2\alpha s}{3\pi} \int_{4m_\pi^2}^{\infty} \frac{1}{s'-s}\left[\frac{\tilde{R}(s')}{s'} - \frac{\tilde{R}(s)}{s}\right] ds'. \tag{44}$$

(B)corresponding $q\bar{q}$ to the quasi-free fermion pairs the calculation is same as for the lepton pairs, but use the electric charge and constituent mass of quark, and the multiply the correction factor of strong interaction 1.2 and color factor 3,

$$\delta_{vac}^{q\bar{q}} = 1.2 \cdot 3 \cdot \frac{2\alpha}{\pi}[\frac{4}{9}f(x_u) + \frac{1}{9}f(x_d) + \frac{4}{9}f(x_c) + \frac{1}{9}f(x_b)], \quad x_q = 4m_q^2/s. \tag{45}$$

Obviously, method (A) contains the contributions of resonant states, and dose not in (B). The numerical results show that the above two methods give similar results in continuous region. In resonant region, $\delta_{vac}^{res}(s)$ is not negligible and sensitive to the resonant parameters $(M, \Gamma^{ee}, \Gamma^{tot})$.

We find the resonant parameters for $\psi(3686)$, $\psi(3770)$, $\psi(4160)$ and $\psi(4416)$ in PDG do not match BES data well. We used two ways to fit BES data in order to obtain the consilient resonant parameters (one is fitting observed cross section, and another is fitting the R value). The differences of the radiative correction factors by these two methods are about $1-2\%$ for most points and $2-3.5\%$ for few points in resonant region.

3 The schemes of radiative correction

In this section, the symbols we used are same as in the previous sections. The definition of the initial state radiative correction factor is as following

$$1 + \delta \equiv \sigma^T(s)/\sigma^\circ(s). \tag{46}$$

The schemes that we have had may be subdivided into roughly two classes: the schemes given by G. Bonneau and F. Martin which are based on the calculations of Feynman figures in PQED, and by E.A. Kureav and V.S. Fardin which is similar to the structure function method in QCD.

3.1 Scheme I

This scheme is based on the calculation by G. Bonneau and F. Martin[8], it gives the general form of the total cross section of e^+e^- annihilation processes,

$$\sigma^T(s) = \sigma^\circ(s)[1 + \frac{2\alpha}{\pi}(\frac{\pi^2}{6} - \frac{17}{36}) + \beta(\frac{13}{12} + \ln k_0)] + \sigma^H(s). \tag{47}$$

The special feature of the reaction channel is characterized by its lowest level cross section $\sigma^\circ(s)$. The calculation of the cross section for hard photon emission σ^H is same as equation in(30), the rest parts correspond to the vacuum polarization and vertex correction and the contribution from the part of k-independent of the bremsstrahlung and the contribution of soft photon. The contributions of self-energy figures of e^+ and e^- disappear after the renormalization. The vacuum polarization in (**??**) was calculated directly by using the skill of Feynman integration, but it contains only the e^+e^- loop, and ignored the $\mu^+\mu^-$, $\tau^+\tau^-$ and $q\bar{q}$ loops. In BEPC energies, the contribution from $\mu^+\mu^-$ $\tau^+\tau^-$ are much less than e^+e^-, but the contribution from $q\bar{q}$ is not alway small (for example, near the resonant peaks).

In formula (47), the soft photon emission term may be rewritten as the integration

$$\beta\sigma^\circ(s) \cdot ln\ k_0 = -\beta\sigma^\circ(s)\int_{k_0}^{1}\frac{dk}{k}, \tag{48}$$

applying the formula (33), we may get a expression which includes the higher levels bremsstrahlung

$$\sigma^T(s) = \sigma^\circ(s)[1 + \frac{2\alpha}{\pi}(\frac{\pi^2}{6} - \frac{17}{36}) + \frac{13}{12}\beta + \delta_\gamma], \tag{49}$$

where,

$$\delta_\gamma = \beta\int_0^{k_m}\frac{dk}{k}k^\beta[(1 - k + \frac{k^2}{2})\frac{\sigma^\circ(s')}{\sigma^\circ(s)} - 1] + k_m^\beta - 1 \tag{50}$$

is the correction factor of bremsstrahlung. Obviously, the integration over k is convergent in $[0, k_m]$. For continuous states, the lightest hadronic state is $\pi^+\pi^-$, therefore

$$k_m = 1 - 4m_\pi^2/s. \tag{51}$$

3.2 Scheme II

This scheme was developed by F.A.Berends and R.Kleiss[9], which was used widely by most Monte Carlo generators. It gives the initial state radiative correction factor and total cross section of $N-$particle production in e^+e^- annihilation via one-photon exchange

$$e^+e^- \Rightarrow \gamma(k)\ +\ N\ hadrons$$

The cross section up tu $O(\alpha^3)$-order was written as the sum of two terms

$$\sigma^T = \sigma^{VSB} + \sigma^H, \tag{52}$$

their expressions are

$$\sigma^{VSB} = \sigma^\circ(s) \cdot [1 + \beta\ \ln(k_0) + \delta_{AR}], \tag{53}$$

the cross section of hard bremsstrahlung σ^H is same as equation (30). The term containing β ln k_0 coming from the contribution of single soft photon emission. δ_{AR} is the correction of the virtual processes (loop-figures) , which may be written as[2]

$$\delta_{AR} = \delta_{vert} + \delta_{vac}^{ee} + \delta_{vac}^{\mu\mu} + \delta_{vac}^{\tau\tau} + \delta_{vac}^{had}, \tag{54}$$

where the vertex correction

$$\delta_{vert} = \frac{2\alpha}{\pi}(\frac{3}{4}\log\frac{s}{m_e^2} - 1 + \frac{\pi^2}{6}). \tag{55}$$

If we use the exponentiation property for soft bremsstrahlung $\beta \ln k_0$ in formula (53), we may get the expressions which were used by Crystal Ball group in its experiments[11]

$$\sigma^T(s) = \sigma^\circ(s)(1 + \delta_{AR} + \delta_\gamma), \tag{56}$$

where, term δ_γ is same as in scheme I.

3.3 Scheme III

According to the point of view of quantum field theory, the initial e^+ and e^- always accompany with the photon emission and reabsorb (mainly in small angle direction), and therefore e^+ and e^- are surrounded by real and virtual cloud of photon. If we ignore the influence of transverse momentum we may use the distribution functions $D_{e^+}(x_1, s)$ and $D_{e^-}(x_2, s)$ to describe the remaining fraction longitudinal momentum x_1 and x_2 of e^+ and e^-.

Based on the QCD practice, the calculations for the process of e^+ and e^- annihilation through single photon to produce some final state are similar to the Drell-Yan process. Applying the general conclusion of KLM theorem about the divergence of mass and infrared factorization, the total cross section may be expressed in terms of the lowest level cross section and the longitudinal momentum distribution function . This scheme is universal to the processes with an arbitrary for $e^+e^- \Rightarrow \mu^+\mu^-, \tau^+\tau^-$, hadrons. In the leading logarithmic approximation, the total cross section reads [12][13][14]

$$\sigma^T(s) = \int\int_{x_1+x_2>2} dx_1 dx_2 D_{e^+}(x_1,s) D_{e^-}(x_2,s) \tilde{\sigma}^\circ(x_1x_2s), \tag{57}$$

where,

$$\tilde{\sigma}^\circ(s) = \frac{\sigma^\circ(s)}{|1-\hat{\Pi}_2(s)|^2}, \tag{58}$$

$\sigma_0(s)$ is the lowest level cross section. We may rewrite $\sigma^T(s)$ in another convenient form by introducing variable $x = 1 - x_1x_2$

$$\sigma^T(s) = \int_0^{x_{max}} dx \frac{\sigma^\circ(s')}{|1-\hat{\Pi}_2(s')|^2} F(x,s), \tag{59}$$

$F(x,s)$ is called the radiator, which includes the corrections of initial vertex, multi-photon emission and the pair-conversion $\gamma \rightarrow e^+e^-$. Here, the quantity x in formula does not in general has the meaning of the fraction of the energy carried away by the photon and by the e^+e^- pairs. The identification of x with this energy fraction is valid at small x, but for $x \sim 1$ it is violated in the α^2 order. At present, there is not a Monte Carlo generator which may simulate the processes corresponding to this theory.

3.4 The effective radiative correction factor

In the previous sections, $(1+\delta)$ is the theoretical value of radiative correction, but its observed values $(1+\delta_{obs})$ in experiment is dependent on the hadronic efficiency. Hence, we should consider the product of radiative correction and efficiency as a factor. The hadronic cross section observed in experiment is

$$\sigma^T_{obs} = \frac{N_{had}}{L}. \tag{60}$$

We may image that R value can be calculated in following two different steps:

$$\sigma^T = \frac{\sigma^T_{obs}}{\bar{\epsilon}} \quad \Rightarrow \quad R = \frac{N_{had}}{\sigma^\circ_{\mu\mu} L\bar{\epsilon}\cdot(1+\delta)}, \tag{61}$$

or

$$\sigma^\circ_{obs} = \frac{\sigma^T_{obs}}{1+\delta_{obs}} \quad \Rightarrow \quad R = \frac{N_{had}}{\sigma^\circ_{\mu\mu} L\epsilon(0)(1+\delta_{obs})}. \tag{62}$$

It should has by comparing the two above formulas

$$\bar{\epsilon}(1+\delta) \approx \epsilon(0)(1+\delta_{obs}). \tag{63}$$

But in fact, we find they are different in values. The reasons are the real radiative corrections are influenced by detecting efficiencies, and the corresponding lowest cross section should be replaced by the effective one, i.e.

$$\sigma^{\circ}(s) \Rightarrow \sigma^{\circ}_{obs}(s) \equiv \epsilon(0)\sigma^{\circ}(s), \quad \sigma^{\circ}(s') \Rightarrow \sigma^{\circ}_{obs}(s) \equiv \epsilon(k)\sigma^{\circ}(s'). \tag{64}$$

But it is not the case for virtual corrections even though the experimental factors are under considerations, the lowest cross section related to virtual correction no need multiply the efficiency. The efficiency of virtual radiation is same as one of non-radiation and soft radiation in Monte Carlo. In general, $\epsilon(0) > \bar{\epsilon}$ and $\epsilon(k)$. The expression $\bar{\epsilon}(1+\delta)$ means that the efficiency for virtual processes have also $\bar{\epsilon}$, it is not reasonable. The reasonable method is to put only the real radiative efficiencies in the term of which corresponding to bremsstrahlung, i.e. $\delta_\gamma \Rightarrow \delta_{\gamma obs}$. So we use $\epsilon(0)(1+\delta_{obs})$ instead of $\bar{\epsilon} \cdot (1+\delta)$. According to the replacement in (64), we may change the theoretical cross section into the corresponding observed cross section, and then get the effective radiative correction factor

$$\frac{\sigma^{T}_{obs}(s)}{\sigma^{\circ}_{obs}(s)} \equiv 1+\delta_{obs} \tag{65}$$

In the scheme III, the variable x is not the energy fraction carring by one photon, there is not the corresponding variable in present Monte Carlo simulations. Evenmore, the real and the virtual radiative corrections are included in the factor $F(x,s)/|1-\hat{\Pi}_2|^2$, instead of being expressed as the sum of each correction like did in scheme I and II. Therefore, scheme III can only give the theoretical value $(1+\delta)$, can not give the effective observed value $(1+\delta_{obs})$.

3.5 About the final state radiative corrections

The final state radiative corrections do not change the c.m. energy, but affect the cross section by a factor. The sum over all hadronic states cancels the contributions from leading logarithmical terms, the remaining is a factor[3] $(1+3\alpha Q^2/\pi^2)$ which is very close to 1. In other side, our understanding to hadronic structures is very rough, we do not nee to pursue the electromagnetic correction at the present experiment precision.

4 The numerical results and discussions

The radiative correction is a "small" correction to the lowest level cross section. To analysis the lowest cross section combined with the setting in experiments in our calculations. There are abound of resonances in BEPC energy region, the interference between them and the energy-dependence of the total width will behavior as the change of radiative correction with the energies. The selected hadron samples contains the events produced in higher level processes. Comparing with the lowest level cross sections, the radiative effects make the structures of the total cross section to be weaker. In other word, the structure of lowest level cross section (i.e. R value) will be significant after the contributions from higher level processes were deducted. The resonant parameters in PDG do not macth BES data well. In our calculations, we fit the resonant parameters $(M, \Gamma^e, \Gamma^{tot})$ of $\psi(3770)$, $\psi(4040)$, $\psi(4160)$ and $\psi(4416)$ globally with two methods (one fits the observed cross sections, and another fit the R values). The interferences of them and the energy-dependence of the total withs were considered in fit.

At the energies just above the thresholds of the resonances (especially for J/ψ and ψ), the radiative correction factors become large. This is due to the bremsstrahlung makes the effective energies locate at the resonant peaks and enhance the cross section and case is contrary at the energies of lower threshold.

The differences of radiative correction factors are smaller than 1% for the continuous hadronic states, and about $1-2\%$ in resonant region if we compare their theoretical values only.

If we treat the differences of the above three schemes as an estimation of the systematic error in theories, under the condition of using the same $\tilde{R}(s)$ values and resonant parameters and σ°_{res}, errors of $(1+\delta)$ are smaller than 1%, and about $1-2\%$ in resonant region. For a fixed scheme, we estimate uncertainty of $(1+\delta)$ from the following aspects: (1)the errors of inputing $\tilde{R}(s)$; (2)the errors of input resonant parameters; (3)the errors of hadronic efficiencies $\epsilon(k)$ estimated in Monte Carlo; (5)errors in numerical integration. The total errors caused by the reasons mentioned above are about $2 \sim 2.5\%$. In the calculations for $R99$ data, we used the scheme II.

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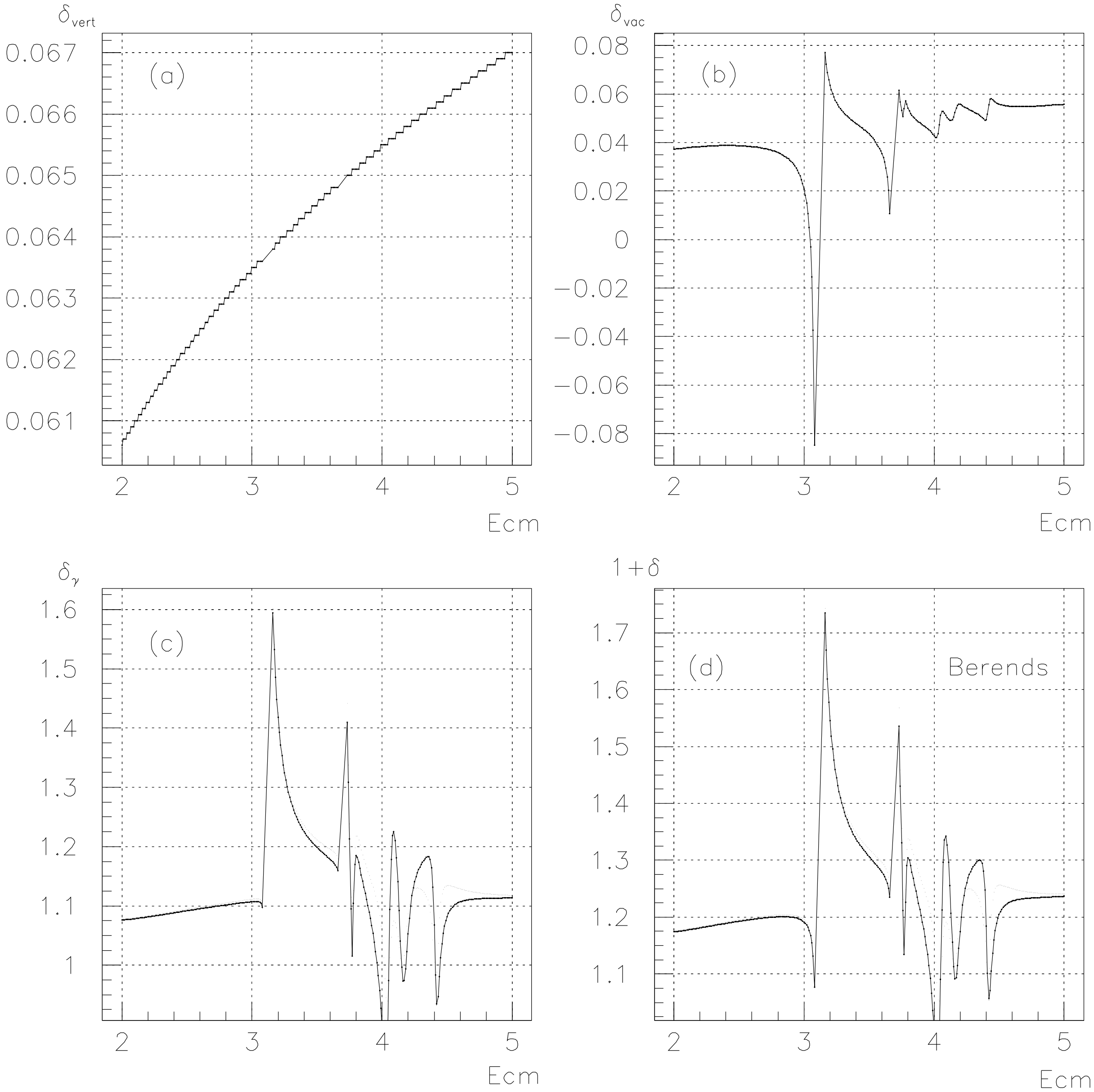

图 3 : The theoretical values of radiative correction by scheme II: (a) for the vertex correction; (b) for the vacuum polarization; (c) for bremsstrahlung; (d) for the total values.

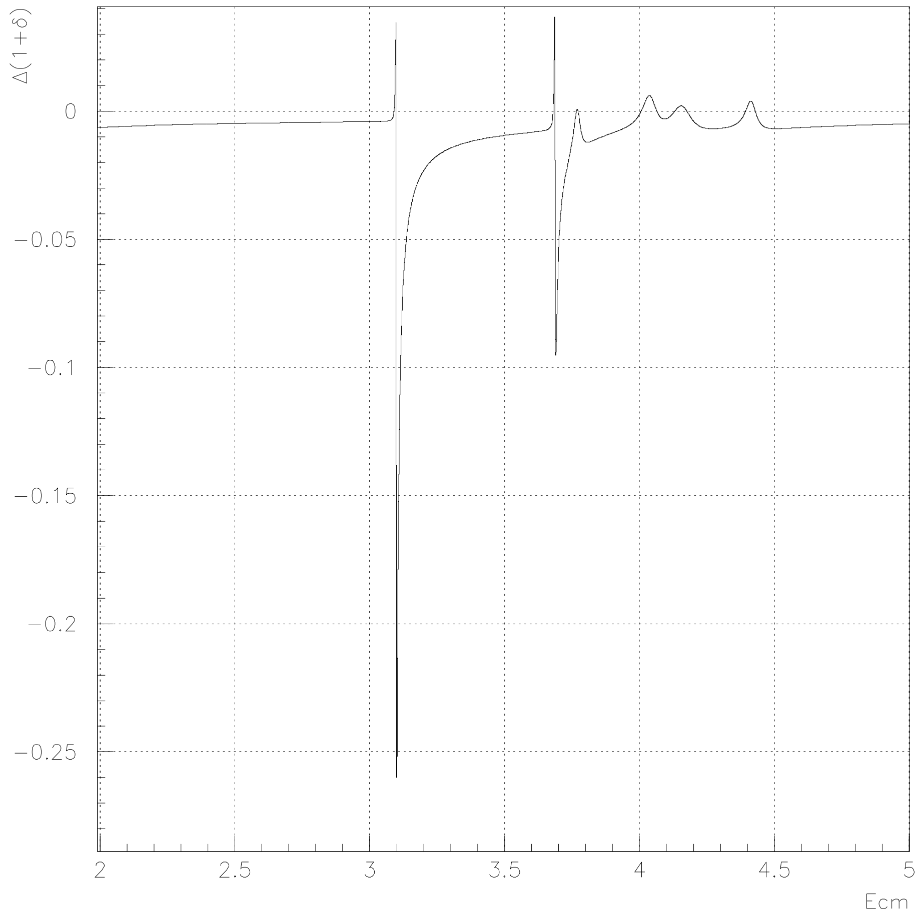

图 4 : The systematical errors of radiative correction factor (theoretical values) which are estimated by the scheme II and scheme III

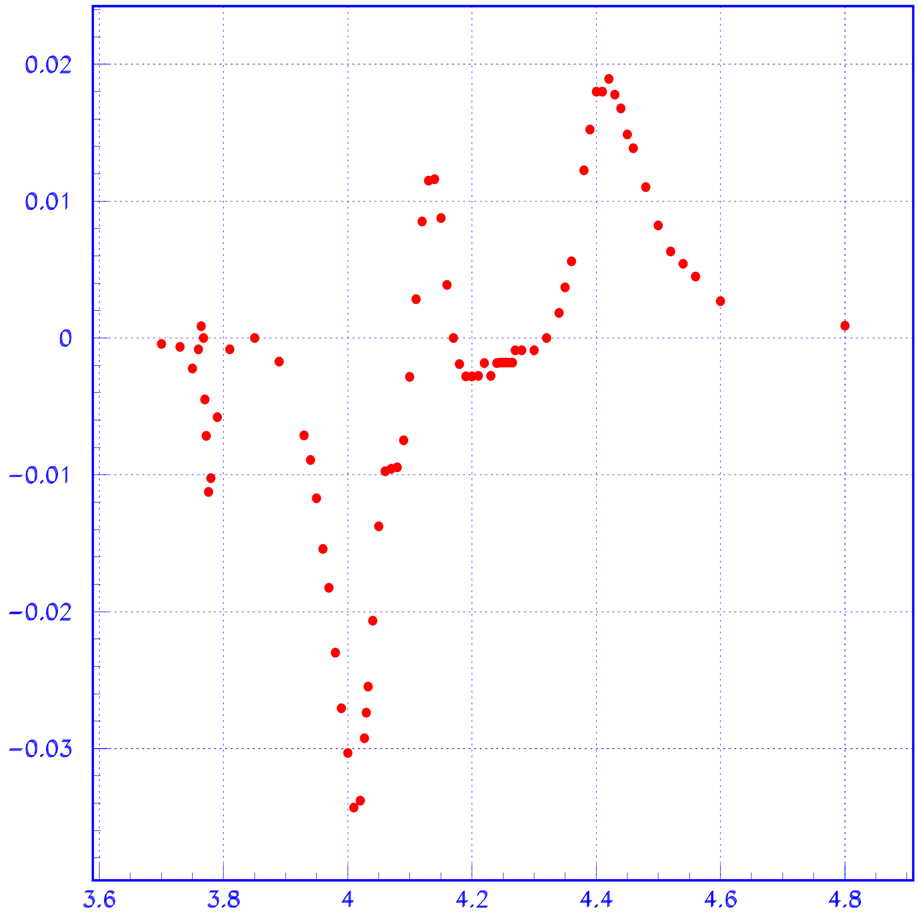

图 5： The change of estimated systematical errors of radiative correction factor due to the uncertainty of resonant parameters with the energies (GeV) in resonant region.