Initial State Radiative Correction in R Measurement at BES

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Abstract

Four typical schemes of the initial state radiative correction in e^+e^- collision through single photon annihilation were studied, and the numerical results were calculated in the energy region $0.3-10$ GeV. For the energy points we are planning to scan with BESII at the energies between $2-5$ GeV, the differences of the radiative correction factors $(1+\delta)$ are about $1\% \sim 2\%$. The uncertainty of $(1+\delta)$ is estimated to be about 1% for a certain scheme.

1 Introduction

To measure the R value, the ratio between the cross section of the hadron production of the e^+e^- annihilation via single photon and that for the muon pair as shown in eq. 1

$$R = \frac{\sigma^{\circ}_{tot}(e^+e^- \to \gamma^* \to hadrons)}{\sigma^{\circ}(e^+e^- \to \gamma^* \to \mu^+\mu^-)}, \tag{1}$$

the radiative corrections should be taken into account to obtain the tree level cross section σ°_{tot} from the experimental measured one.

From the experiment point of view, The value of R is determined from the number of observed hadronic events (N^{obs}_{had}) by the relation

$$R = \frac{N^{obs}_{had} - N_{bg} - \sum_l N_{ll} - N_{\gamma\gamma}}{\sigma^0_{\mu\mu} \cdot L \cdot \epsilon_{had} \cdot \epsilon_{trg} \cdot (1+\delta)}, \tag{2}$$

where $(1+\delta)$ is the initial radiative correction (ISR) factor, which represents the contributions of all higher-order Feynman diagrams in the measured cross section σ^{obs}_{had}.

The final state radiative corrections can be ignored by the KLN cancellation theorem[1]. The correction is estimated to be less than one percent [2].

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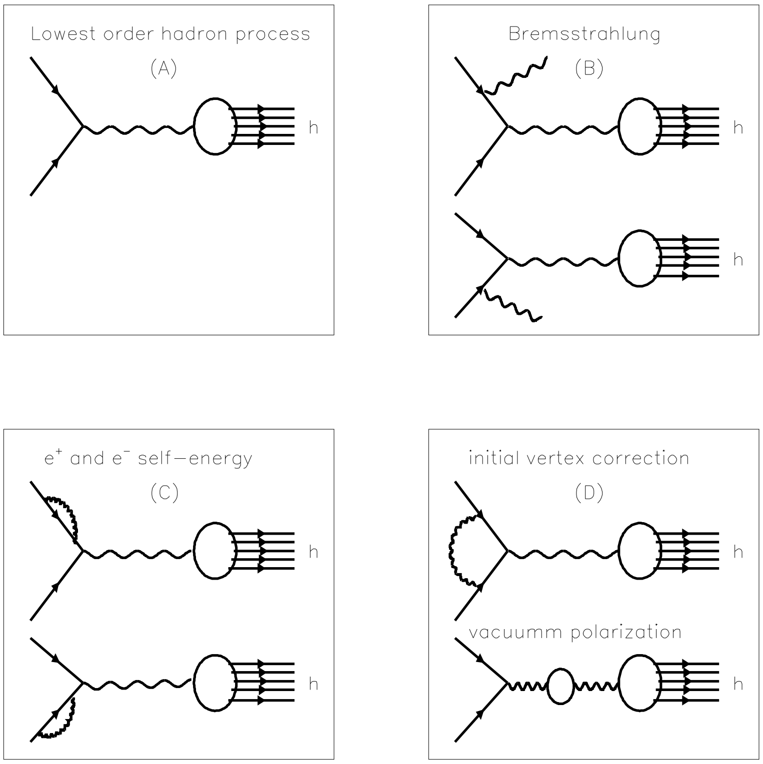

Figure 1: Feynman diagramss of $e^+e^- \Rightarrow$ hadrons processes. The later three diagrams are the higher order processes, whose contributions are represented by radiative correction factor δ.

2 Calculation schemes of ISR

At the begining of this section, we describe briefly some notations used in the calculation of radiative correction. R values concern hadronic processes

$$e^+e^- \Rightarrow \quad continuous \ / \ resonant \quad states$$

The inclusive cross sections may be written as

$$\sigma^\circ(s) = \sigma^\circ_{con}(s) + \sigma^\circ_{res}(s) \qquad (lowest \quad order) \tag{3}$$

$$\sigma^T(s) = \sigma_{con}(s) + \sigma_{res}(s) \qquad (total) \tag{4}$$

i.e., the hadronic cross section may be presented as the resonant part add to continuum one. In the calculation, the former in the continuum is given by

$$\sigma^0_{con}(s) = \tilde{R}(s) \cdot \sigma^0_{\mu\mu}(s) \tag{5}$$

where, $\tilde{R}$ is the R value that the resonant contributions were removed. The values of $\tilde{R}$ may use the one from fitting the experimental data in all energy region, or take the experimental values from hadron production threshold $2m_\pi$ to 1.5 GeV, while take $R_{QCD}(s)$ in > 1.5 GeV where PQCD is regarded to be valid. In the energy region of $\sqrt{s} > 1.5$ GeV, the predictions of $R_{QCD}(s)$ are in agreement with the averaged expermental data, and the errors of the former are smaller than that of the latter. There are abundant resonant states in BEPC energy region. The contributions from the following 20 resonances are considered

$$\rho,\ \omega,\ \phi,\ J/\psi,\ \psi',\ \Upsilon,\ \Upsilon',\ \cdots \quad \text{etc.},$$

The resonant cross section, as shown in figure 2, are parametrized as Breit-Wigner form

$$\sigma^0_{res} = \sum_j \sigma^b_j(s) = \sum_j \frac{12\pi\Gamma^{ee}_j\Gamma^{tot}_j}{(s-M^2_j)^2+(M_j\Gamma^{tot}_j)^2}. \tag{6}$$

The resonant parameters were taken from PDG. If the measurement is made at far from the resonance peak, Breit-Wigner cross section can be simplified as (except for wide resonance ρ)

$$\sigma^b(W) \Rightarrow 12\pi^2\frac{\Gamma_{ee}\Gamma_h}{\Gamma_T M_R}\delta(W^2-M^2_R). \tag{7}$$

In addition, beam energy-spread should be considered. The expected relation between the experimentally observed effective resonant cross section and the Breit-Wigner cross section is given by a convolution integral

$$\tilde{\sigma}_{res}(s) = \int_{-\infty}^{+\infty} ds'\sigma_{res}(s')G(\sqrt{s}-\sqrt{s'}) \tag{8}$$

where $G(\sqrt{s}-\sqrt{s'})$ is the distribution of machine energy spread, which has the Gaussian form

$$G(\sqrt{s}-\sqrt{s'}) = \frac{1}{\sqrt{2\pi}\Delta}exp\left[-\frac{(\sqrt{s}-\sqrt{s'})^2}{2\Delta^2}\right], \tag{9}$$

where Δ is the beam energy spread in the c.m. energy. For BEPC $\Delta = 1.88\times10^{-4}\sqrt{s}$ (in GeV).

The observed hadronic cross sections are not identical with the physical one, the detection efficiency for the hadronic events should be considered. The observed effective cross sections are

$$\sigma^0_{obs} = \varepsilon^0_{had}\cdot\sigma^0(s) \quad (lowest\ order) \tag{10}$$
$$\sigma^T_{obs} = \bar{\varepsilon}_{had}\cdot\sigma_T(s) \quad (total) \tag{11}$$

ε^0_{had} and $\bar{\varepsilon}_{had}$ are the detection efficiencies with and without initial state radiation.

Radiative corrections are approximate perturbative calculation and depend on the renormalization skill. In this work, we calculated the radiative correction using the following four widely used schemes. The numerical results were compared and some discussions were made.

2.1 Scheme I

This scheme, which is used by most Monte Carlo generators, was developed by F.A. Berends and R.Kleiss[4]. It gives the initial state radiative correction factor and total cross section of $N-$particle production in e^+e^- annihilation via one-photon exchange

$$e^+e^- \Rightarrow \gamma(k) \ + \ N \ \ hadrons$$

In this scheme, there is a so called k_0 problem. It has been proved that for each fixed order $O(\alpha^n)$, QED is renormalizable if all corresponding Feynman diagrams are combined, by which the ultrared divergences of loop diagrams can be removed. The PQED corrections in both virtual and real photon emission are infrared divergencies. The divergence in real photon emission comes from photon with zero energy, and in virtual photon part from photon with zero mass and energy. One skill to solve this problem is to introduce a small photon mass λ in order to keep the theory being Lorentz invariant and to regularize the infrared divergences. An arbitrary number k_0 describs separately the infrared behavior for soft bremsstrahlung and the nonfactorizing cross section for hard bremsstrahlung photons. The infrared divergence in the virtual corrections is just an artefact of perturbative theory. Since the zero energy and soft photon are not observable, one can define

$$\begin{aligned} \sigma^S &\equiv \sigma_0 + \sigma^V + \sigma^{SB} && (12) \\ \sigma^H &\equiv \sigma^{HB}. && (13) \end{aligned}$$

Here λ does not appear in σ^S due to the cancellation. Both σ^S and σ^H are finite and are related to k_0, a non-physics quantity used to categorize soft and hard photon from bremsstrahlung. The radiative correction should not depend on the choice of k_0 within reasonable limits. The following requirements are considered

$\star$ upper limit: this is imposed by phenomenological considerations. Only those real bremsstrahlung photon with $k > k_0$ were observed, and those with $k < k_0$ have to be "lumped" together with the virtual photons. This means that k_0 should at least be smaller than the resolution power of the detector;

$\star$ lower limit: as $k_0 \to 0$, $\sigma^S \to -\infty$, The chosen k_0 value must ensure $\sigma^S > 0$.

The cross section up to $O(\alpha^3)$ order was written as the sum of two terms

$$\sigma_{obs} = \sigma_{obs}^{VSB} + \sigma_{obs}^{HB} \tag{14}$$

where

$$\begin{aligned} \sigma^{VSB} &= \epsilon(0)\sigma_0(s) \cdot (1 + \beta\ ln\ k_0 + \delta_{AR}) && (15) \\ \sigma^{HB} &= \int_{k_0}^{k_{max}} dk \epsilon(k) \frac{\partial \sigma}{\partial k}. && (16) \end{aligned}$$

β is the equivalent radiator

$$\beta = \frac{2\alpha}{\pi}[ln\frac{s}{m_e^2} - 1], \tag{17}$$

and

$$\frac{\partial \sigma}{\partial k} = \frac{\beta}{k}(1 - k + \frac{1}{2}k^2)\sigma_0(s') \tag{18}$$

is the spectrum of real photon emission. The term $\beta\ ln\ k_0$ comes from the contribution of soft photon emission, and $\epsilon(k)$ is the detection efficiency for the hadronic events with radiative photons of energy k estimated by Monte Carlo as shown in figure 3 for $E_{cm} = 3.55\ GeV$. The upper limit k_{max} in integral is determined by the requirement that the reduced effective c.m. energy of e^+e^- system must be larger than the hadron production threshold.

$$k_{max} = 1 - s_{th}/s = 1 - 4m_\pi^2/s \tag{19}$$

The vertex and vacuum polarization correction factors may be written as[1]

$$\delta_{AR} = \delta_{vert} + \delta_e + \delta_\mu + \delta_\tau + \delta_{had}, \tag{20}$$

The vertex correction term is expressed as

$$\delta_{vert} = \frac{2\alpha}{\pi}(\frac{3}{4}ln\frac{s}{m_e^2} - 1 + \frac{\pi^2}{6}). \tag{21}$$

The vacuum polarization terms may be represented by dispersion relation

$$\delta_{vac}^X = \frac{s}{2\pi^2\alpha}Re\int_{s_{th}}^{\infty}\frac{\sigma_{ee\to X}(s')ds'}{s-s'-i\epsilon}, \quad (X = e^+e^-, \mu^+\mu^-, \tau^+\tau^-, hadrons) \tag{22}$$

Vacuum polarization is a virtual process, the hadronic cross section $\sigma_{ee\to had}$ should not multiplied by hadronic efficiency in this integral. For the lepton loop ($x = 4m_l^2/s$), the dispersive integral gives the analytical result

$$\delta_{vac}^{l\bar{l}}(x) = \frac{2\alpha}{\pi}\left\{-\frac{5}{9} - \frac{x}{3} + \frac{\sqrt{1-x}(2+x)}{6}ln\left[\frac{1+\sqrt{1-x}}{1-\sqrt{1-x}}\right]\right\} \qquad (x \le 1) \tag{23}$$

$$\delta_{vac}^{l\bar{l}}(x) = \frac{2\alpha}{\pi}\left\{-\frac{5}{9} - \frac{x}{3} + \frac{\sqrt{x-1}(2+x)}{3}tan^{-1}\frac{1}{1-\sqrt{x-1}}\right\} \qquad (x > 1) \tag{24}$$

for the hadronic term, the cross section reads

$$\sigma_{had}(s) = \sigma_{had}^{con}(s) + \sigma_{had}^{res}(s) \tag{25}$$

therefore, the corresponding correction factors are

$$\delta_{vac}^{had}(s) = \delta_{vac}^{res}(s) + \delta_{vac}^{con}(s) \tag{26}$$

where,

$$\delta_{vac}^{res}(s) = \sum_j \frac{6\Gamma_j^{ee}}{\alpha M_j}\cdot\frac{s\left[s - M_j^2 + \Gamma_j^{tot}M_j\pi^{-1}ln\left(s/M_j^2\right)\right]}{\left(s-M_j^2\right)^2 + \left(M_j\Gamma_j^{tot}\right)^2} \tag{27}$$

$$\delta_{vac}^{con}(s) = -\frac{2\alpha s}{3\pi}\int_{4m_\pi^2}^{\infty}\frac{1}{s'-s}\left[\frac{\tilde{R}(s')}{s'} - \frac{\tilde{R}(s)}{s}\right]ds' \tag{28}$$

The results of fixed $O(\alpha^n)$ PQED suffer from the so called k_0 problem. In the point of view of experiment, k_0 can be used as a scale of energy resolution in order to distinguish between soft and hard photons. The resolution for the photon at BES is about 50 $\sim$ 80 MeV, we may choose $0.01 < k_0 < 0.05$ $(k \equiv E_\gamma/E_b)$, from which the uncertainty of the theoretical calculation is estimated to be $< 1\%$.

2.2 Scheme II

The results obtained by fixed order PQED are effective for hard photon $k > k_0$. For very soft photon $k < k_0$, the effects of higher orders are important. If the contributions from arbitrary number soft photon emissions are included, the soft cross section has the leading exponentiation property, i.e., they lead to the introduction of factor k^β in the spectrum of bremsstrahlung

$$\frac{\partial\sigma}{\partial k} \sim \sigma_0 \times \frac{\beta}{k} \times k^\beta \tag{29}$$

its integral over $0 \sim k_{max}$ is convergent

$$\beta \int_0^E \frac{dk}{k} (\frac{k}{E})^\beta = 1$$

the k_0 problem does not appear clearly.

The standard way in which the experimentalists tend to treat the initial state radiative correction is based on the method introduced by G. Bonneau and F. Martin[3], which gives a general formula for the radiative corrections (in α^3) to processes of e^+e^- into any final state (except e^+e^-). The original formula can be written as

$$\begin{aligned} \sigma(s) &= \sigma_0(s)\{1 + \frac{2\alpha}{\pi}(\frac{\pi^2}{6} - \frac{17}{36}) + (\frac{13}{12} + ln\frac{k_0}{E})\beta & (30) \\ &+ \beta \int_{k_0}^{k_{max}} \frac{dk}{k}(1 - \frac{k}{E} + \frac{k^2}{2E^2})\frac{\sigma_0(s')}{\sigma_0(s)}\}, & (31) \end{aligned}$$

The term of integral in parentheses {} corresponds to radiative correction of hard photon emission, the other parts correspond to the vertex form factor due to vacuum polarization and mass renormalization and soft photon emission as well as a part of the external Bremsstrahlung which is independent of k. According to the exponentiation property, the observed cross section may be rewritten as

$$\begin{aligned} \sigma_{obs}(s) &= \epsilon(0)\sigma_0(s)\{1 + \frac{2\alpha}{\pi}(\frac{\pi^2}{6} - \frac{17}{36}) + \frac{13}{12}\beta & (32) \\ &+ \beta \int_0^{k_m} \frac{dk}{k} \left(\frac{k}{E}\right)^\beta [(1 - \frac{k}{E} + \frac{2k^2}{E^2})\frac{\epsilon(k)\sigma_0(s')}{\epsilon(0)\sigma_0(s)} - 1]\}. & (33) \end{aligned}$$

We note that the vacuum polarization does not represent by dispersion relation, the correspondence of vacuum polarization, as compared to scheme I, is weeker. It leads the significant difference at the vicinity of resonant states.

2.3 Scheme III

This scheme was used by Crystal Ball Collaboration[5]. It is very similar with the scheme I, but the exponentiation is used in real photon emission correction term. The cross section measured in experiment reads

$$\sigma_{had}^{obs} = \sigma_{had}^{\circ} \cdot \overline{\varepsilon}_{had} \cdot (1 + \delta), \tag{34}$$

where, $\overline{\varepsilon}_{had}$ is the average detection efficiency for the hadronic events which is estimated by Monte Carlo. More practical, radiative effect in $\overline{\varepsilon}_{had}$ is moved into the radiative correction factor

$$\overline{\varepsilon}_{had}(1 + \delta) => \epsilon(0)(1 + \delta_{obs}), \tag{35}$$

where $\epsilon(k)$ is the efficiency with radiative energy k, δ_{obs} is observed effective radiative correction. The basic formula of the radiative correction factor is

$$\begin{aligned} 1 + \delta_{obs} &= 1 + \delta_{vert} + \delta_{vac} & (36) \\ &+ \beta \int_0^{k_{max}} \frac{dk}{k}(k)^\beta[(1 - k + \frac{1}{2}k^2)\frac{\epsilon(k)\sigma_0(s')}{\epsilon(0)\sigma_0(s)} - 1]\}, & (37) \end{aligned}$$

δ_{obs} is presented as the sum of the corresponding terms

$$\delta_{obs} = \delta_{vert} + \delta^{e}_{vac} + \delta^{\mu}_{vac} + \delta^{\tau}_{vac} + \delta^{res}_{vac} + \delta^{con}_{vac} + \delta_{\gamma obs}. \tag{38}$$

The meaning of each term is noted in its index. The initial vertex and vacuum polarization correction terms are the same as those in scheme I, and the real photon emission term is the same as in scheme II, in which the exponentiation is adopted.

2.4 Scheme IV

According to QCD practice, the process of single photon annihilation may be regarded as a Drell-Yan process. One can introduce structure functions for the photons to write the total cross section as a convolution of lowest one at reduced (effective) energy and the part describing photon emission. This scheme is universal to processes with an arbitrary final state, e.g. $e^+e^- \Rightarrow \mu^+\mu^-, \tau^+\tau^-$, hadrons.

In the leading logarithmic approximation[6],

$$\sigma(s) = \int\int_{x_1+x_2>2} dx_1 dx_2 D_e(x_1,s) D_e(x_2,s) \tilde{\sigma}(s x_1 x_2) \tag{39}$$

where

$$\tilde{\sigma}(s) = \frac{\sigma_0(s)}{|1-\Pi(s)|^2}, \tag{40}$$

$\Pi(s)$ is vacuum polarization factor,

$$2\Pi(s) = \sum_X \delta^X_{vac}(s). \tag{41}$$

Introduce a variable $x = 1 - x_1 x_2$,

$$\sigma(s) = \int_0^{x_{max}} dx \frac{\sigma_0(s')}{|1-\Pi(s')|^2} F(x,s), \tag{42}$$

where radiator $F(x,s)$ includes radiative corrections, both initial vertex and multi-soft/hard photons emission, and the contribution from converse production of e^+e^- pairs $\gamma \to e^+e^-$. Here, the quantity x in formula does not in general have the meaning of the fraction of the energy carried away by the photon and by the e^+e^- pairs. The identification of x with this energy fraction is valid at small x, but for $x \sim 1$ it is violated in the α^2 order. At present, there is not a Monte Carlo generator which may simulate the processes corresponding to this theory.

The presentation of this scheme is not unique. In reference [7], the factor

$$\frac{F(x,s)}{|1-\Pi(s')|^2} \tag{43}$$

in [6] was replaced by

$$f(x,s) = (1+\delta_{vp})(1+K)[\beta k^{\beta-1}(1+\delta_1+\delta_2) - \beta(1-k/2)] \tag{44}$$

i.e., the vacuum polarization was expanded as a series and take only the first two terms. They are identical with high precision only $\Pi(s)$ is small, in other word, it is not correct in the vicinity of resonance. This requirement is not always satisfactory, as one can see from figure 4. The later will suppresses the long-tail effect in the high threshold of the resonance.

3 Numerical results

In this section, the numerical results of the total hadron production cross section $\sigma_T(s)$ and the radiative correction factor $(1+\delta)$ using the four schemes described previously are presented for the energy region $0.3 \sim 10.3$ GeV, and the comparisons are made.

In the calculations, the Breit-Wigner cross sections of resonances (except for ρ) are approximated by δ function for the energy points far from the resonant peaks. The influence of beam energy spread should be considered near the vicinity of the resonances, which is not our case.

Figure 5-14 are the cross sections, radiative correction factors from four schemes utilized. The detailed description for each figure is made in the corresponding caption.

For energy points we are interested in, the differences of the total cross sections $\sigma_T(s)$ by these schemes are less than 1%, and of the radiative correction factors $(1+\delta)$ are about $1\% \sim 2\%$. The However, differences are more significant in the vicinity of the resonant peaks.

The errors of $(1+\delta)$ and hence $\sigma_T(s)$ for each scheme is less than 1 (2) the experimental errors of R values, or the uncertainty of R_{QCD} in theory; (3)the error of ϵ_{had}. The numerical integral errors is at the level of 0.1% and therefore is negligible.

The total error of $(1+\delta)$ at our measured energy points of the R scan performed in Spring of 1998 are about $1\% \sim 3\%$, if one take the difference among the four schemes as systematical error.

Figure 15-18 show the differnces of the total hadron production cross sections and of the radiative correction factors.

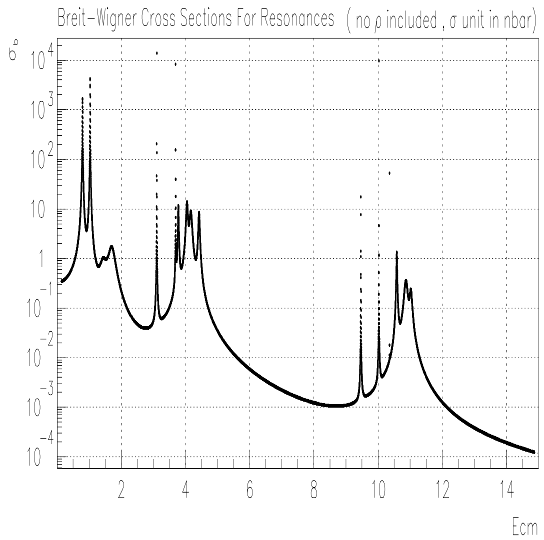

Figure 2: Resonant state cross section by Breit-Wignerform

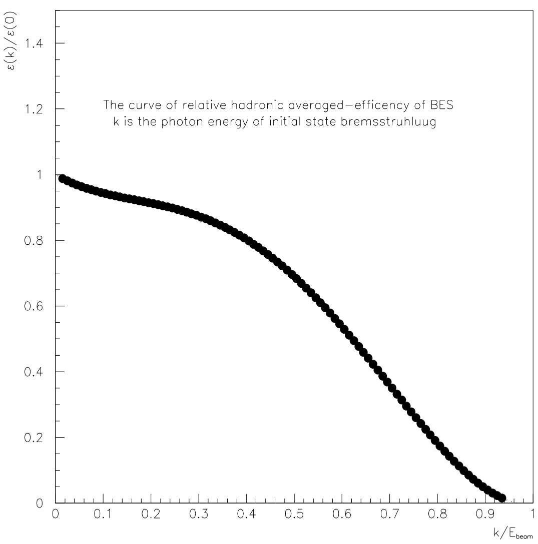

Figure 3: Hadronic efficiency with real photon emission of energy k at $E_{cm} = 3.55$ GeV

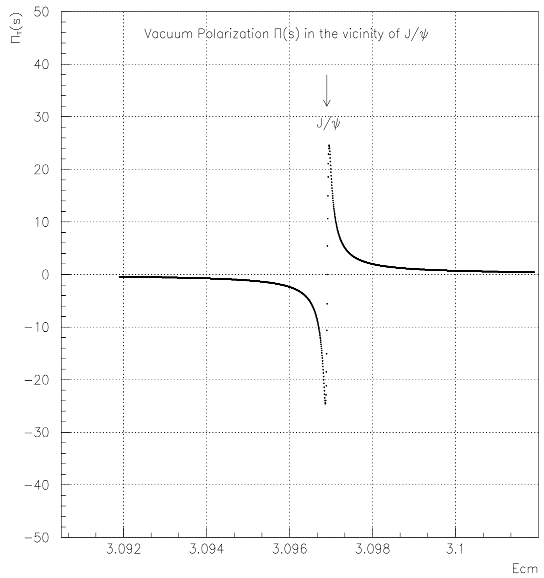

Figure 4: Vacuum polarization factor $\Pi(s)$ in the vinicity of J/ψ, the similar case for the other resonances

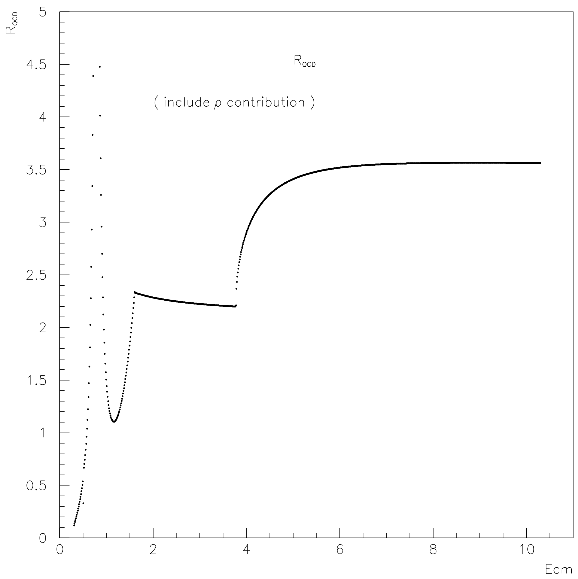

Figure 5: R_{QCD} value used in calculation of radiative correction

4 Summary

We have calculated the radiative correction using four different schemes in 0.3 $\sim$ 10 GeV. The difference among these schemes are compared and the following conclusions have been reached:
a) In the continuum region, the four schemes used are in consistent within 1%.
b) At the very near resonance region, particularly near J/ψ and $\psi\prime$, the four schemes differ about $1\% - 3\%$.
c) For a certain scheme, the total error of the radiative correction is about $(0.5 - 1.0)\%$.

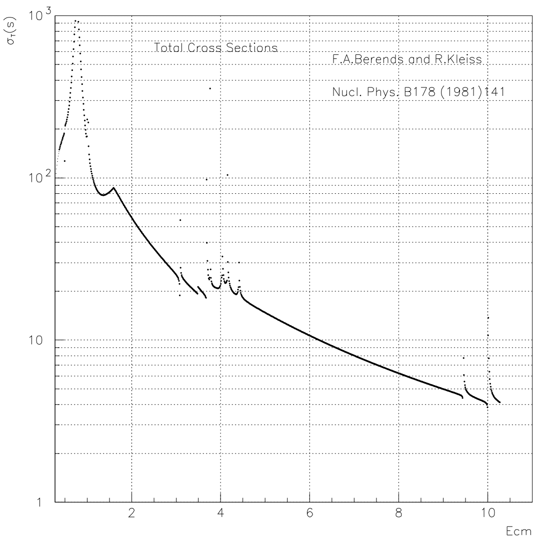

Figure 6: The total cross sections given by scheme I

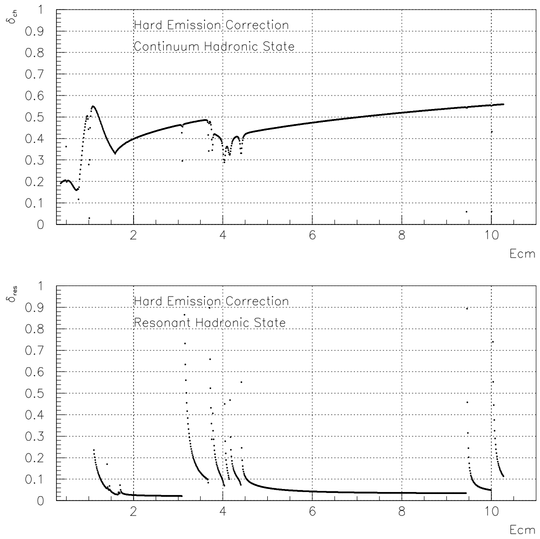

Figure 7: The radiative correction terms in scheme I

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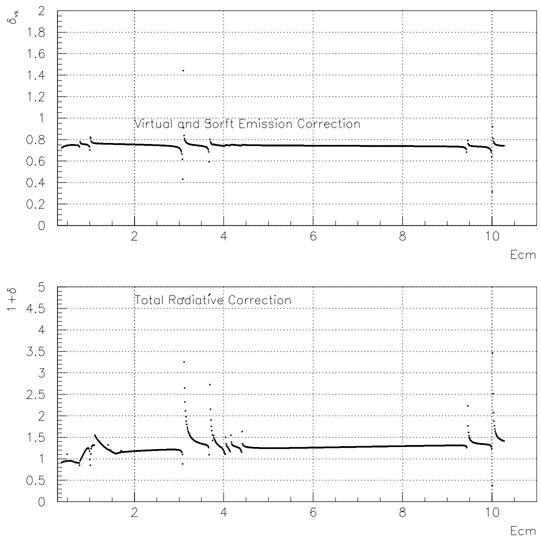

Figure 8: The radiative correction terms in scheme I

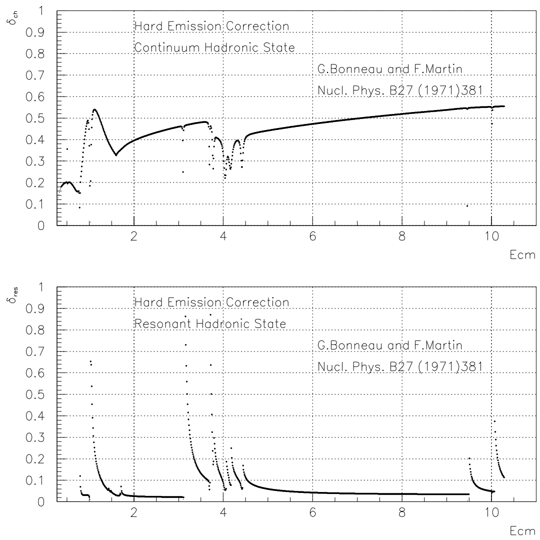

Figure 9: The soft and hard radiative correction terms in scheme II

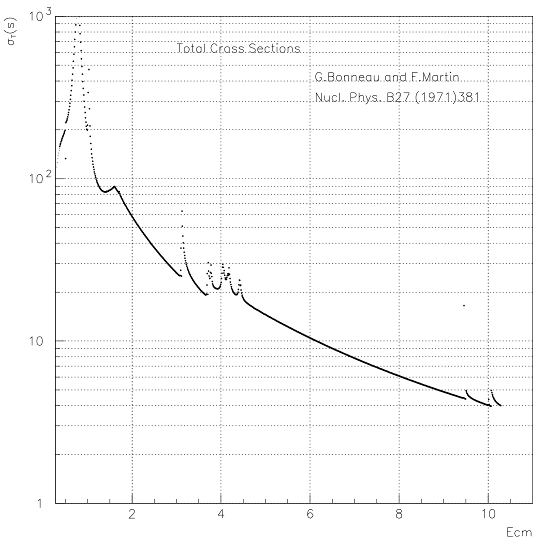

Figure 10: The total hadronic cross section given by scheme II

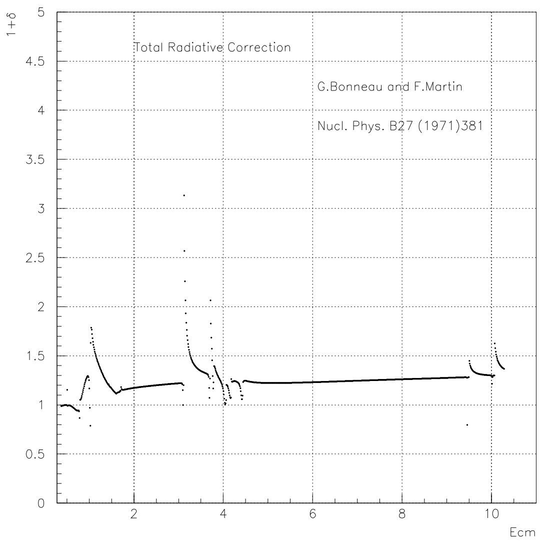

Figure 11: The total radiative correction factor given by scheme II