The Improvement Of The Calculation Method Of The Error-Matrix In MDC Track Fit

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- **The Calculation Of The Error-Matrix In The MDC Track Fit**
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ABSTRACT

This note reports the processing of the modifications for the calculation method of the error-matrix in MDC track fit. Three method of the calculation of fit weight are described. finally got a better fit weight and be used in the fit codes. The results from the physics checking indicates the method solved the error matrix problem better. But more correction will be required further due to the quality of real data changed complexly.

1 The Calculation of the Error-Matrix in the MDC tracking

1.1 The Track Description of the charged particles in the magnetic field

The motion equation of the charged particles in the magnetic field is as following:

$$M\ddot{X} = \frac{Q}{C}\dot{X} \times \vec{B} \tag{1}$$

in a given system and a special magnetic field, suppose the $B_x = B_y = 0$, B_z=C, The orbit of the charged particles can be described as:

- **X=X$_0$+Rsin**$(\phi - \phi_0)$
- **Y=Y$_0$+Rcos**$(\phi - \phi_0)$
- **Z=Z$_0$+S**$\cdot \sin\lambda$
- ϕ=ϕ_0+**S**$\cdot cos\lambda$/**R**
- $\lambda = \lambda_0$
- **R=[P/(x$\cdot$ q $\cdot$ B)] $\cdot$ cos**λ

where x_0, y_0 and z_0 is the original position of particles, R is the radius of track circus in the x-y plane. λ is the angle of the track direction and the x-y plane(the dip angle), ϕ_0 is the angle from the x-axis original point of particles in the x-y plane.

In order to simplify the calculation, selection a track system ($\hat{n}$, $\hat{b}$ and $\hat{h}$), track parameters: $\phi, k = 1/P_\perp = 1/(pcos\lambda), s = tg\lambda, \xi, \eta$, after the particle moving ΔS in the space, and the momentum of the particle from P change to P', then the momentum and direction of track is described:

- $\vec{P'} = \vec{P} + \rho[sin\theta\hat{b} + (1 - cos\theta)\hat{n} + \theta \cdot tg\lambda_H \cdot \hat{h}]$
- $\vec{t'} = (\hat{t'} \cdot \hat{b})\hat{b} + (\hat{t'} \cdot \hat{h})\hat{n} + (\hat{t'} \cdot \hat{h})\hat{h}$
- $\vec{t'} = cos\lambda_H(cos\theta\hat{b} \pm sin\theta\hat{n} + tg\lambda_H\hat{h})$

1.2 Track Fit and Calculation Of The Error Matrix

In the track space helix fit, the optimize function χ^2 defined as:

$$\chi^2 = \sum_{i=1}^{N} \frac{1}{\sigma_i^2}[d_i^m - d_i(\alpha_\mu)]^2 \tag{2}$$

where the d_i^m and the $d_i(\alpha_\mu)$ is the distance of the closest approach by measured and predicted respectively, the α_μ(1-5) is the track parameters, the N is the number of hit wires. The $1/\sigma_i^2$ is the fit weight of each wires.

The least-square fit calculate the parameters of the equation to yield the minimum value for the χ^2, that is:

$$\frac{\partial \chi^2}{\partial \alpha_\mu} = -2\sum_{i=1}^{N} \frac{1}{\sigma_i^2}[d_i^m - d_i(\alpha_\mu)]\frac{\partial d_i}{\partial \alpha_\mu} = 0 \tag{3}$$

After derivation the χ^2 can be calculated by:

$$\chi^2 = \sum_{i=1}^{N} \frac{1}{\sigma_i^2}(d_i^m - d_{0i})^2 + \sum_{i=1}^{\nu} Y_\nu(\Delta\alpha) \tag{4}$$

Where the $\Delta\alpha$ expressed in matrix is:

- $[\Delta\alpha] = -[G^{-1}][Y]$

  The Y_u and $G_{\mu\nu}$ is defined as:

- $Y_u = \sum_{i=1}^{\mu} \frac{(d_{0i} - d_i^m)}{\sigma^2} \frac{\partial d_i}{\partial \alpha_\mu}$

- $G_{\mu\nu} = \sum_{i=1} \frac{1}{\sigma^2} \frac{\partial d_i}{\partial \alpha_\nu} \frac{\partial d_i}{\partial \alpha_\mu}$

In the real calculation, the χ^2 and error matrix is calculated by following equation:

$$\chi^2 = \sum_{i=1}^{N} \frac{1}{\sigma_i^2}(d_i^m - d_{0i})^2 + \sum_{i=1}^{\nu} Y_\nu(\alpha_\nu - \alpha_{\nu 0}) \tag{5}$$

$$\langle \delta\alpha_\mu \delta\alpha_\nu \rangle = (G^{-1})_{\mu\nu} \tag{6}$$

From equation 5 and 6, we can see the $1/\sigma_i^2$, so called fit weight, is a very important parameters both in track fit and the calculation of the error matrix. In the principle, the spatial resolution of each wire is used as the fit weight in the MDCI at first. It is used also in the R-value measurement. We did not find any problem in the tracking efficiency and the track momentum resolution.

2 The Modifications of Calculation Method Of The Error-Matrix

2.1 The MDC Spatial Resolution From Calibration

For J/ψ data in the beginning of this year, the σ_{xy} is about 200 μm, is used as the fit weight for 40 wires layers. This value come from the distribution of the Δ D= $D_m - D_f$, D_m and D_f is the drift distance of the closed approach by the measurement and fit respectively. The typical plots in some layers is shown in the Fig. 1. This value also can be got from "three wire method"[(t1+t3)/2-t2=2δ], the δ is the wire shift and equal to 700 μm, from Fig. 2(9) we can get the average σ in all layers is about 195 μm (700*24.2/87.16).

Fig.1 The ΔD distribution from the MDC calibration

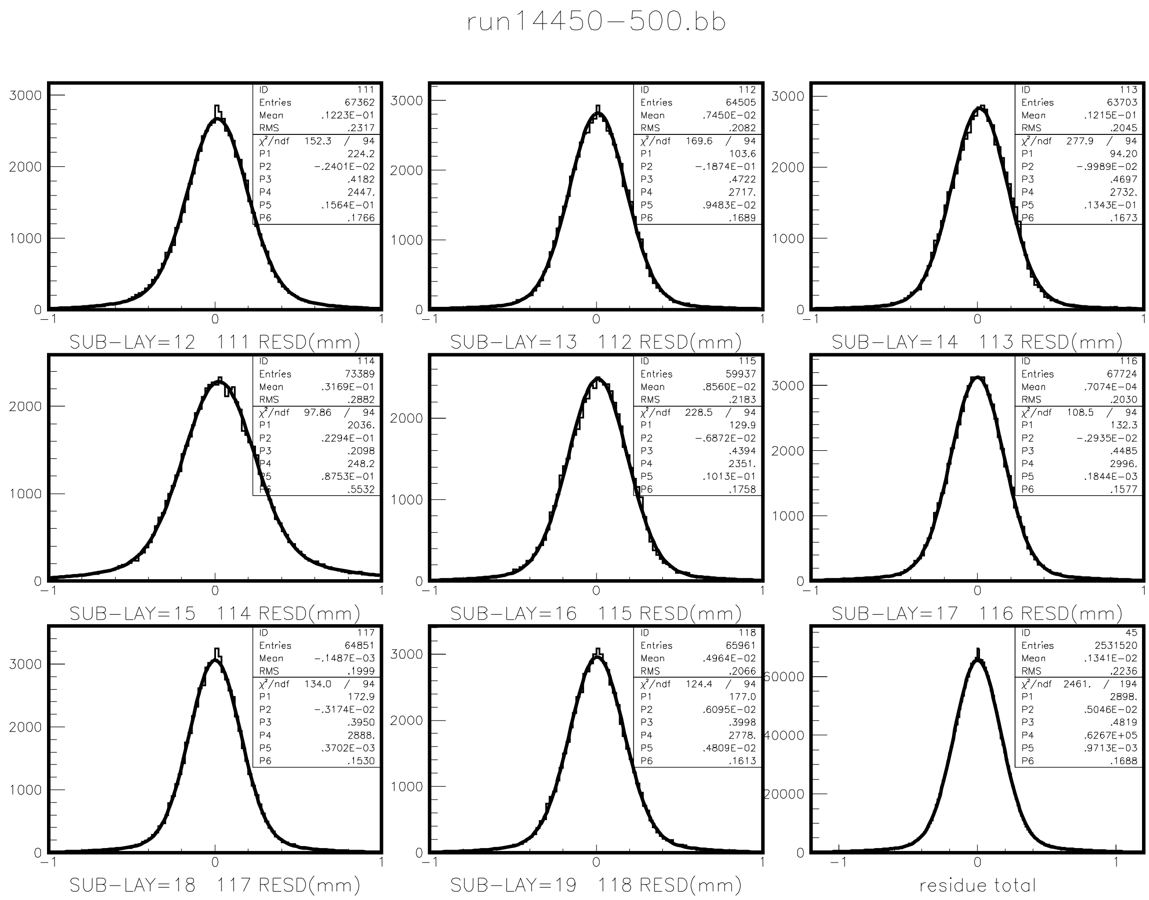
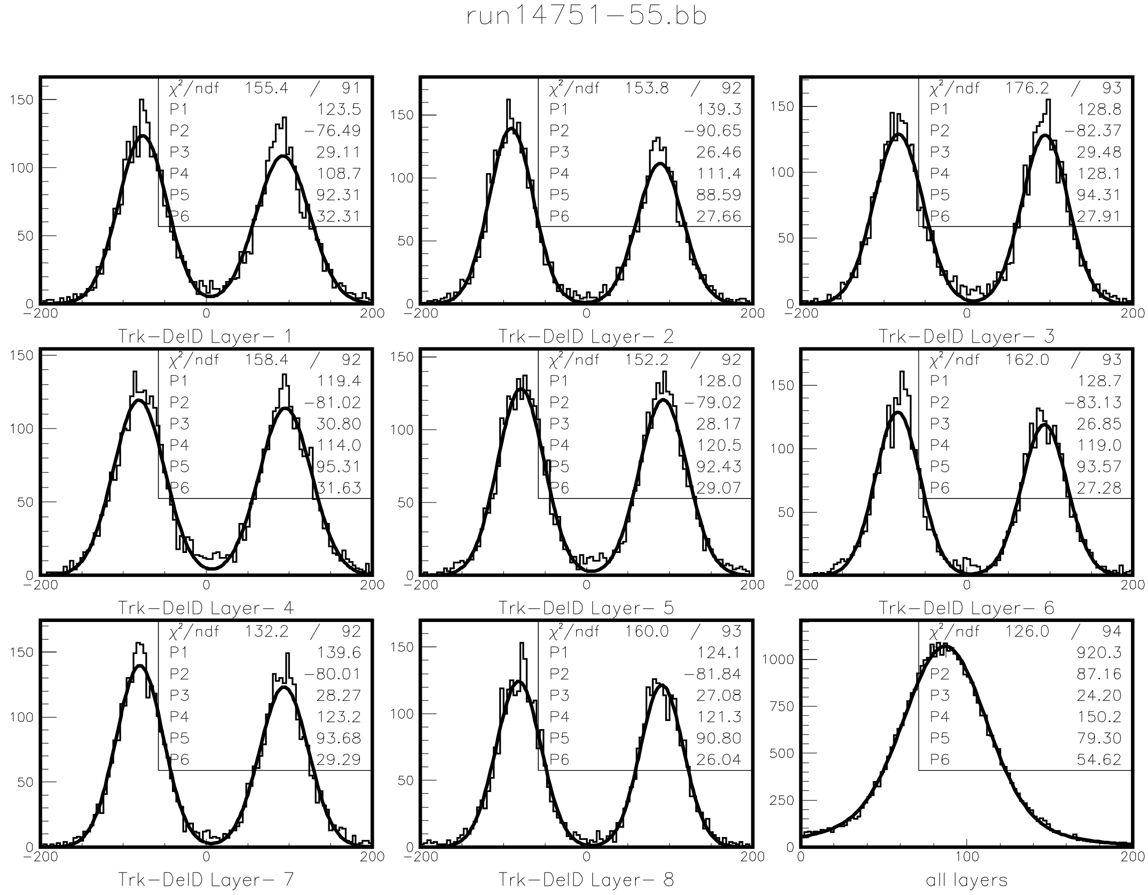


Fig.2 The ΔD distribution by the "three wire method"



2.2 The Spatial Resolution Calculated Layer By Layer

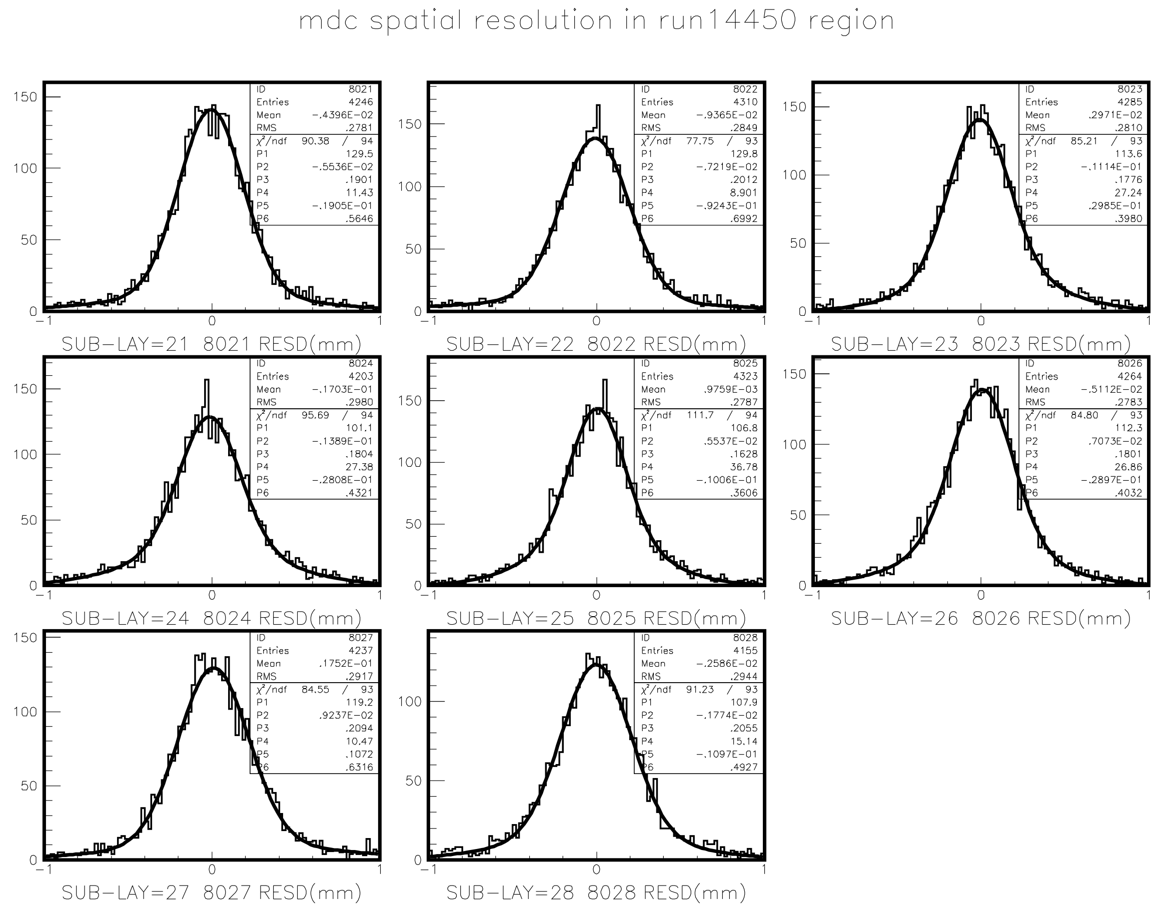
About few hundreds of runs of j/ψ data has been reconstructed by above spatial resolution as the fit weight. After checking using di-muon events, one found that width of 'pull" distribution obtained from Telesis fitting is much big than 1. We also noticed the errors in BESII is less than that of BESI. So, may be the problem is caused by the error matrix which is not correctly calculated.

To solve this problem, we calculation the spatial resolution layer by layer, and set the fit weight for the calculated layer into 999. to avoid that layer join the fit. Then get the spatial resolution for each wire layers, The spatial resolution of first 10 wire layers is shown in following table.

Layer Num.	1	2	3	4	5	6	7	8	9	10
$\sigma_{xy}(\mu m)$	272	261	268	266	274	253	259	295	260	238

The typical plots of the ΔD distribution is shown in Fig.3. Also we try to change other parameters in the track fit codes, for example, increase or reduce to give up the number of “bad points” or change the value of the χ^2 in the fit, but we did not find a good way which can help to solve the error matrix problem.

Fig.3 The ΔD distribution calculated layer by layer



2.3 A New Method to Calculate The Fit Weight

About 2000 run were reconstructed by above σ_{xy} as the fit weight for the physics analysis checking. After doing the telesis, the j/ψ decay into $\phi\pi\pi$ and f_2(1270) is analyzed, the invariant mass plots of the ϕ is worst than that of BESI, but the invariant mass plots of $\pi\pi$ is consistent with that of BESI as shown in the Fig.4 and 5.

Fig.4 the invariant mass plot of $\phi\pi\pi$ in BESI and BESII

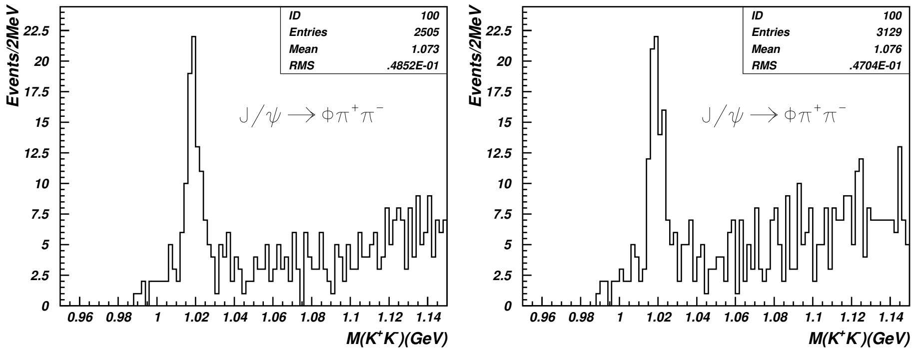
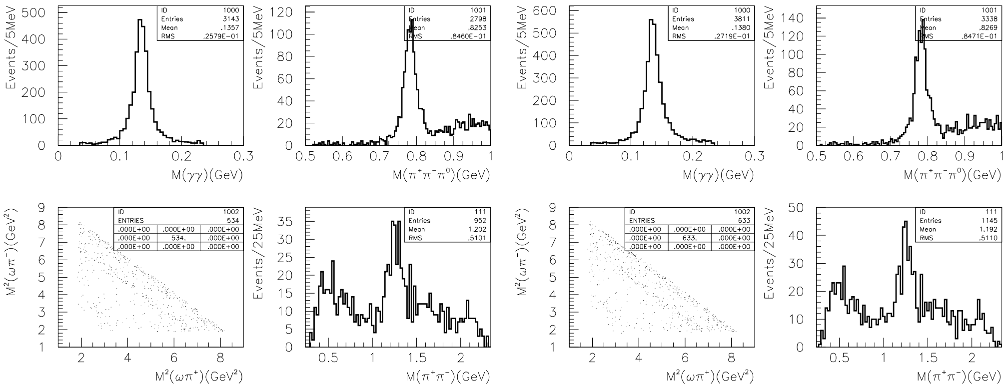


Fig.5 the invariant mass plot of $\pi\pi(\mathrm{f}_2(1270))$ in BESI and BESII



Those plots indicates the plots of (KK) and $(\pi\pi)$ has some different, and the differences of the characteristic of KK and $\pi\pi$ is just the mass and the dE/dx. So we check the relationship of the ΔD versus the Q(dE/dx*distance) as shown in the fig.6, and the distribution of the ΔD to get the spatial resolution in a given Q region as shown in fig. 7. Then get the spatial resolution around the Q as shown in fig.8. This plot shows the spatial resolution varies as Q changing. May be this is the main element of error matrix problem, because we use a constant spatial resolution as the fit weight before. To get Q dependence of σ_{xy} (spatial resolution), we fit the plots in Fig.8 by the function:

$$\sigma_{xy} = A + \frac{B}{\sqrt{Q}} + C \times Q \tag{7}$$

Fig.6 The ΔD versus the Q(dE/dx)

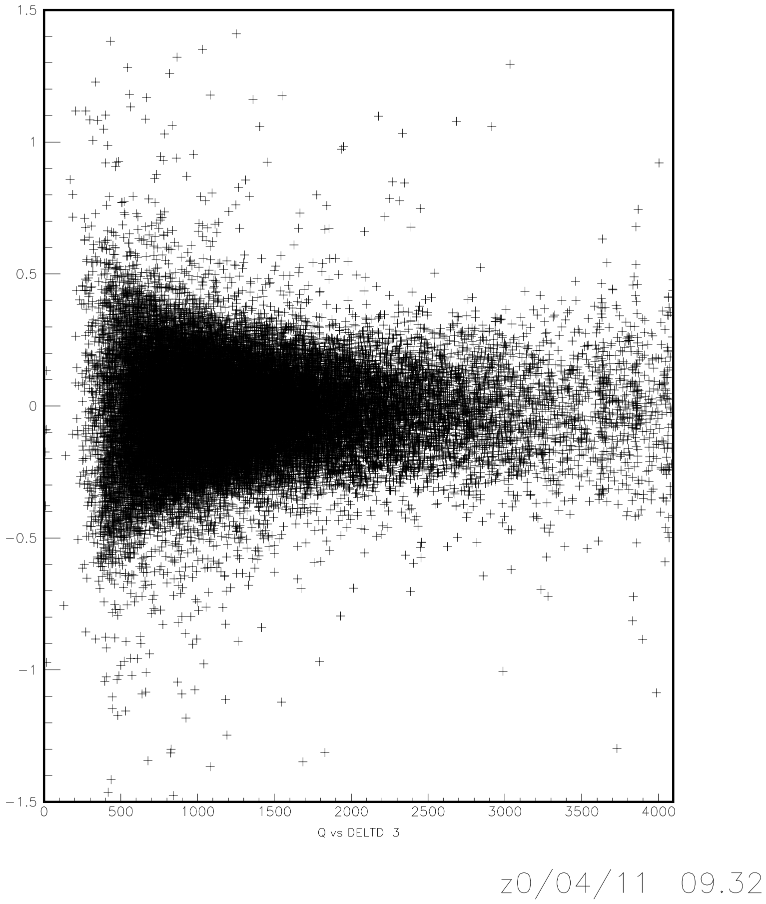
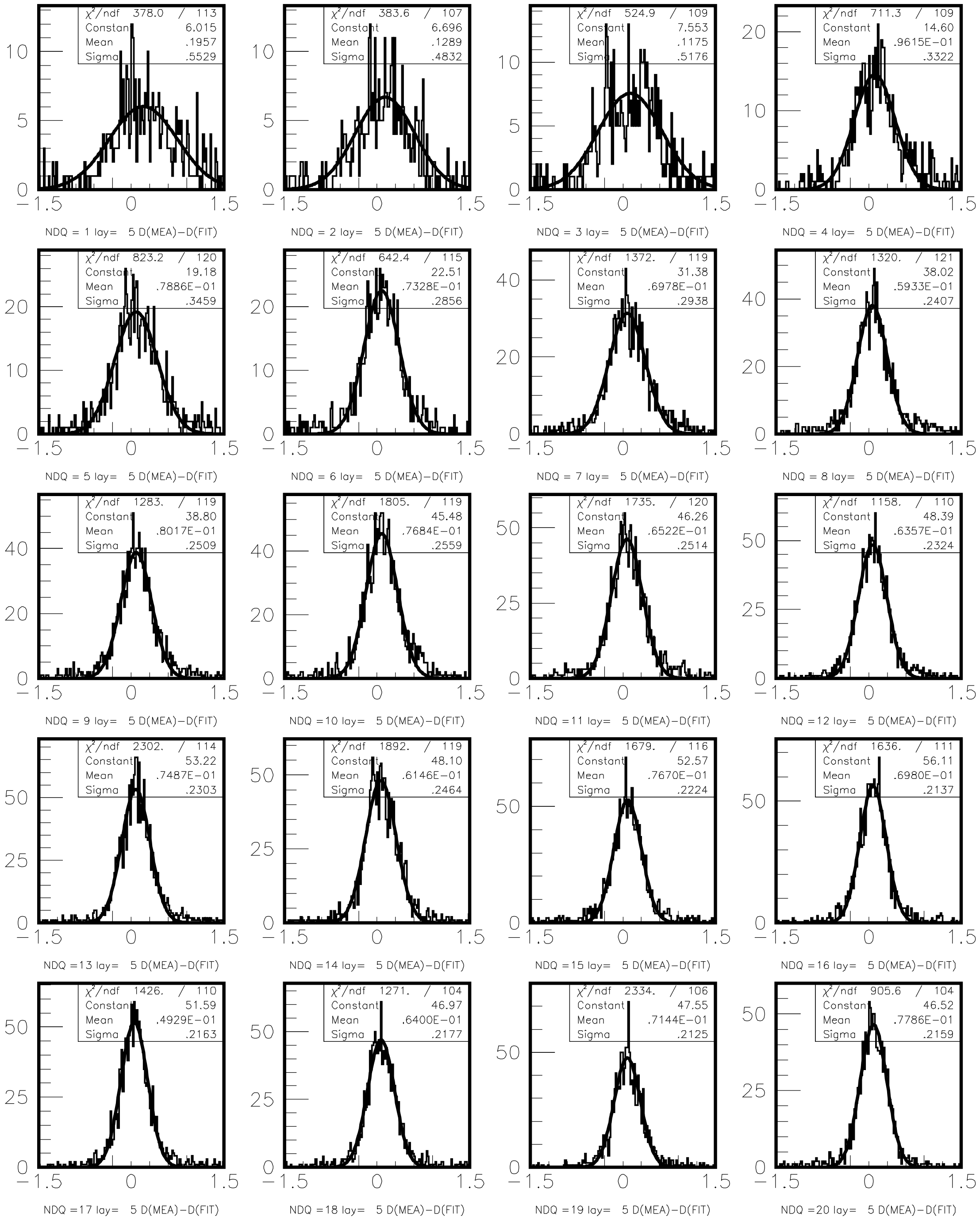


Fig.7 The ΔD distribution in a given Q region



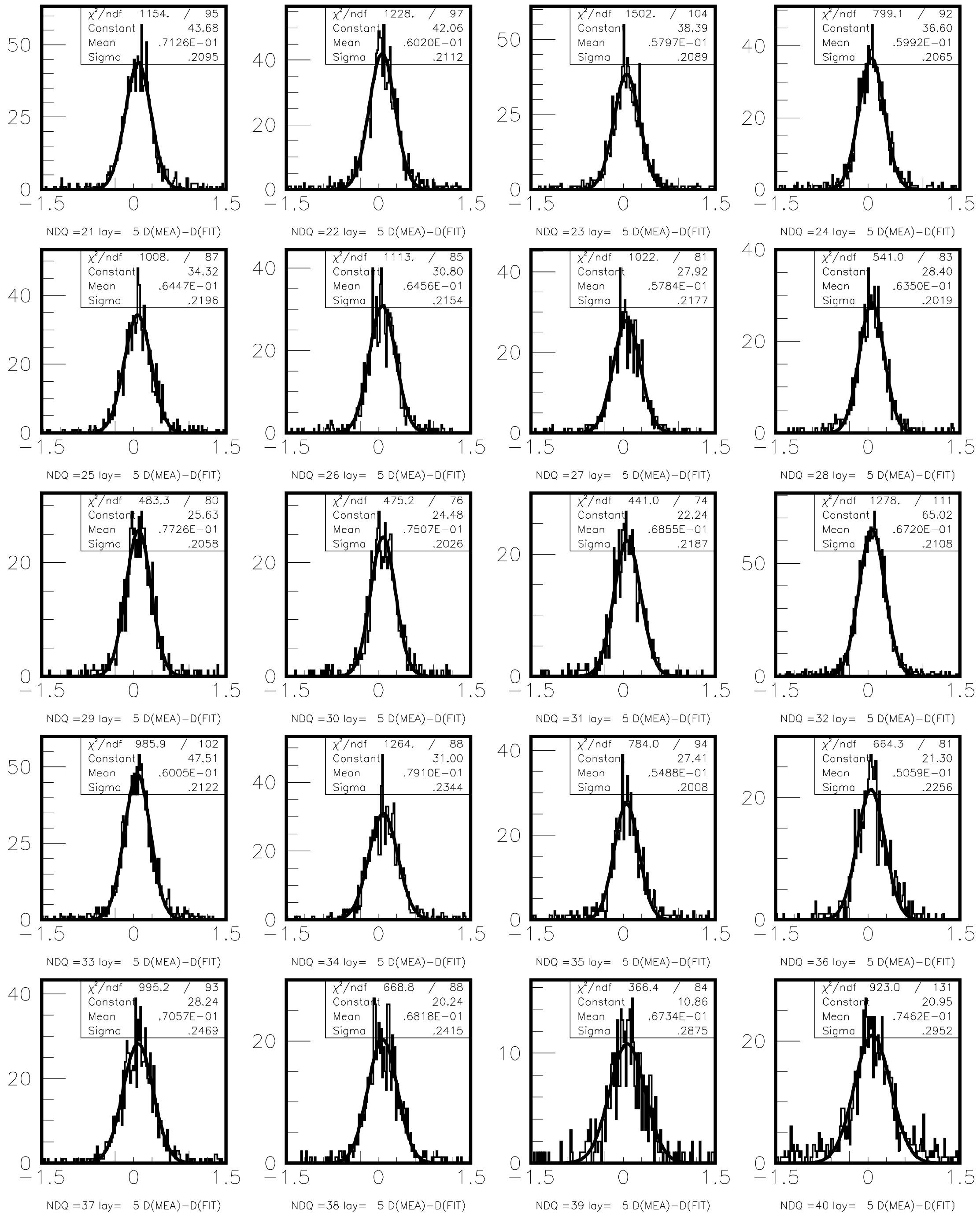
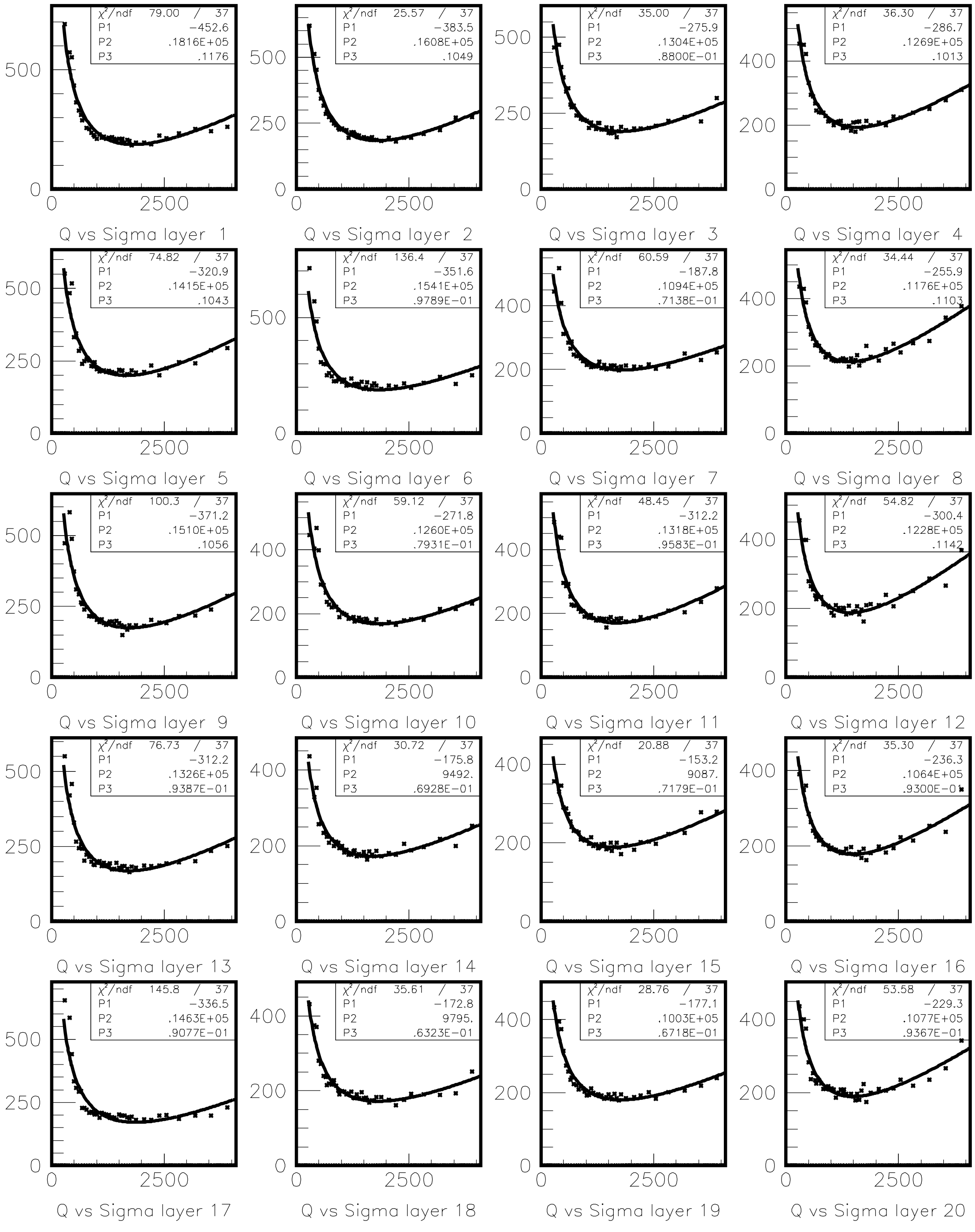
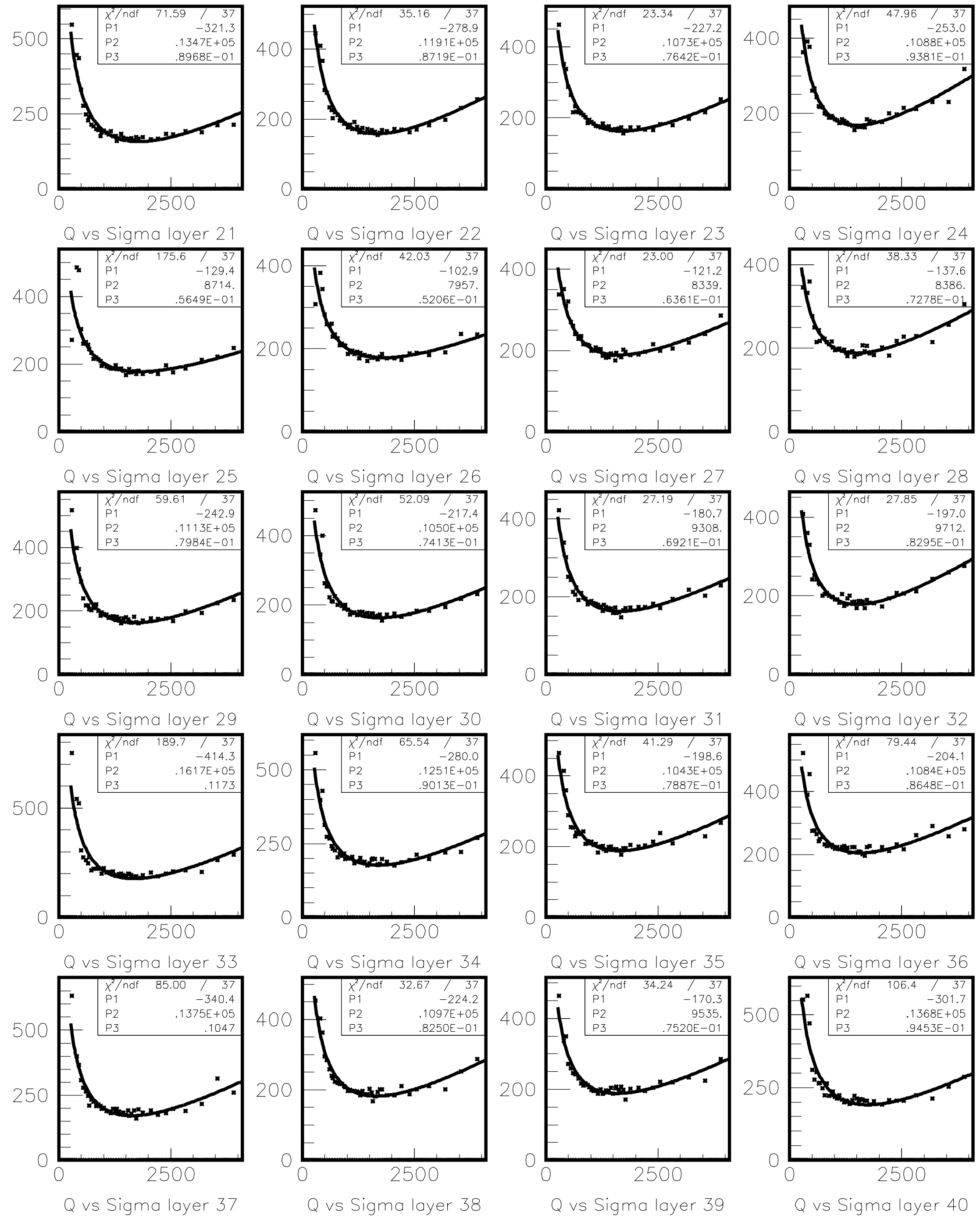

χ²/ndf 1154. / 95
Constant 43.68
Mean .7126E-01
Sigma .2095
NDQ =21 lay= 5 D(MEA)-D(FIT)
χ²/ndf 1228. / 97
Constant 42.06
Mean .6020E-01
Sigma .2112
NDQ =22 lay= 5 D(MEA)-D(FIT)
χ²/ndf 1502. / 104
Constant 38.39
Mean .5797E-01
Sigma .2089
NDQ =23 lay= 5 D(MEA)-D(FIT)
χ²/ndf 799.1 / 92
Constant 36.60
Mean .5992E-01
Sigma .2065
NDQ =24 lay= 5 D(MEA)-D(FIT)
χ²/ndf 1008. / 87
Constant 34.32
Mean .6447E-01
Sigma .2196
NDQ =25 lay= 5 D(MEA)-D(FIT)
χ²/ndf 1113. / 85
Constant 30.80
Mean .6456E-01
Sigma .2154
NDQ =26 lay= 5 D(MEA)-D(FIT)
χ²/ndf 1022. / 81
Constant 27.92
Mean .5784E-01
Sigma .2177
NDQ =27 lay= 5 D(MEA)-D(FIT)
χ²/ndf 541.0 / 83
Constant 28.40
Mean .6350E-01
Sigma .2019
NDQ =28 lay= 5 D(MEA)-D(FIT)
χ²/ndf 483.3 / 80
Constant 25.63
Mean .7726E-01
Sigma .2058
NDQ =29 lay= 5 D(MEA)-D(FIT)
χ²/ndf 475.2 / 76
Constant 24.48
Mean .7507E-01
Sigma .2026
NDQ =30 lay= 5 D(MEA)-D(FIT)
χ²/ndf 441.0 / 74
Constant 22.24
Mean .6855E-01
Sigma .2187
NDQ =31 lay= 5 D(MEA)-D(FIT)
χ²/ndf 1278. / 111
Constant 65.02
Mean .6720E-01
Sigma .2108
NDQ =32 lay= 5 D(MEA)-D(FIT)
χ²/ndf 985.9 / 102
Constant 47.51
Mean .6005E-01
Sigma .2122
NDQ =33 lay= 5 D(MEA)-D(FIT)
χ²/ndf 1264. / 88
Constant 31.00
Mean .7910E-01
Sigma .2344
NDQ =34 lay= 5 D(MEA)-D(FIT)
χ²/ndf 784.0 / 94
Constant 27.41
Mean .5488E-01
Sigma .2008
NDQ =35 lay= 5 D(MEA)-D(FIT)
χ²/ndf 664.3 / 81
Constant 21.30
Mean .5059E-01
Sigma .2256
NDQ =36 lay= 5 D(MEA)-D(FIT)
χ²/ndf 995.2 / 93
Constant 28.24
Mean .7057E-01
Sigma .2469
NDQ =37 lay= 5 D(MEA)-D(FIT)
χ²/ndf 668.8 / 88
Constant 20.24
Mean .6818E-01
Sigma .2415
NDQ =38 lay= 5 D(MEA)-D(FIT)
χ²/ndf 366.4 / 84
Constant 10.86
Mean .6734E-01
Sigma .2875
NDQ =39 lay= 5 D(MEA)-D(FIT)
χ²/ndf 923.0 / 131
Constant 20.95
Mean .7462E-01
Sigma .2952
NDQ =40 lay= 5 D(MEA)-D(FIT)

Fig.8 The spatial resolution around the Q in one wire layer





By this way, we separate the full Q into 40 region for one wire layer, make the ΔD distribution in each Q region to get the $\sigma_i (i = 1 - 40)$ and get the plot of σ versus Q, then get the coefficiency A, B and C in the equation 7. In total we get 120 coefficiency (3×40) from 1600 plots for all layers. Then put this function and coefficiency into fit codes, and make the MDC calibration again. We checked tracking efficiency and the momentum resolution of charged particles, it is almost same as before.

3 The Conclusion

- **After making above modification for the fit weight, the invariant mass plot of J/$\psi \rightarrow \phi\pi\pi$ as shown in Fig. 9 is comparable with BESI. It indicates the error matrix is much reasonable than before.**

- **But the error of the track parameters is strongly depends on the data quality that is changed very complexly for real data, more study and the further correction are required.**

Fig.9 the invariant mass plot of $\phi\pi\pi$ in BESI and BESII